

The Monte Carlo Method

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- Statistical technique
- Model complicated or chaotic systems
- Requires a "good" random number generator

Random Numbers

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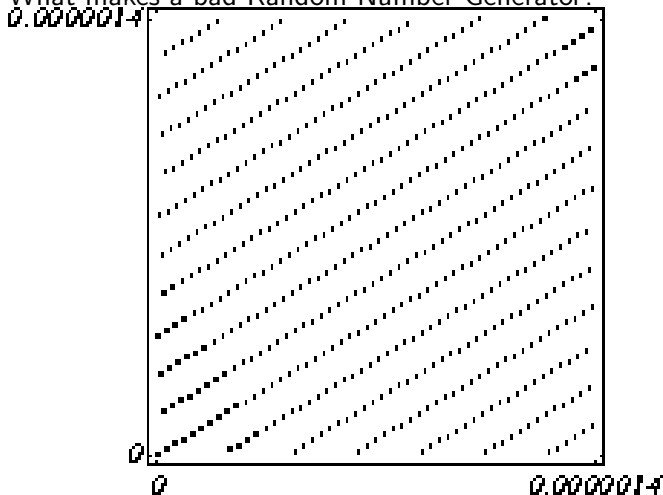
```
int getRandomNumber()  
{  
    return 4; // chosen by fair dice roll.  
             // guaranteed to be random.  
}
```

Randomisation

What makes a bad Random Number Generator?

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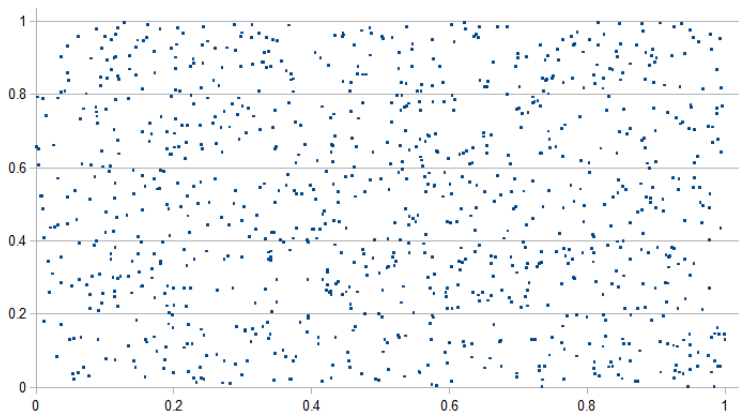


"Good" Random number generator

"Good" Random number generator



A better RNG for simulations



Bias

$$\text{Bias} \propto \mathcal{O}\left(\frac{1}{N}\right)$$

Thermalisation

How can we remove this bias?

- Initial sweeps:

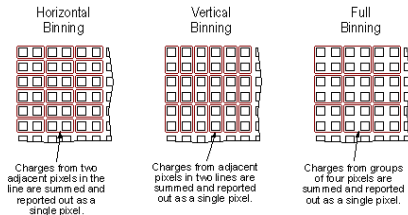
$$\frac{n}{N}$$

- Overall Bias:

$$\frac{1}{\sqrt{N}}$$

Binning

Method of pre-processing to reduce the effect of observation errors.



Jackknife method

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$$\sigma_{f(\bar{x})} = \sqrt{N-1} \sigma_{f^J}$$

Where

$$\sigma_{f^J}^2 \equiv \overline{(f^J)^2} - (\bar{f}^J)^2$$

Questions