Particle Physics

30 January 2012
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20% CA
80% exam

Recommended books:

1. The basics
2. The forces of Nature
   - EM
   - Strong
   - Weak
3. Quarks, Hadrons & colour
4. Experimental methods
5. Open Questions

1: The Basics

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	PH-101	PH-229
Observables	Velocity, forces, momentum, energy	Ditto
Scales	$L \sim \theta(1-10^{21})m$	$10^{-12}-10^{-15}m$
	$M \sim \theta(1 \to 10^{24} \to 10^{30} \to ?)kg$	$10^{-21}(1eV) \to 10^{-25}(proton)kg$
	$V \sim \theta(1)\ ms^{-1}$	$V \sim c = 3 \times 10^{8} ms^{-1}$
Paradigm	Newtonian (aka classical)=human intuition	QM & relativity
Experiments	Human like, billiard balls	Can only use particles themselves to do the interacting & observing. There are no photon sized spanners
How do we study?	Cameras, microscopes, telescopes, spanners	See above: we have to smash particles together to work out what makes them work

2 approaches
"top down"=historical
Long winded
"bottom up" = logical
We start with smallest particles and how everything else can be made from them

NB: Proton has mass of ~1 GeV

Standard model particles

Fermions (=matter particles)

Leptons

Neutrinos	ν_e (small mass)	ν_μ (small mass)	ν_τ (small mass)
Charged Leptons	e^- (1/2 MeV)	μ^- ($10^2 MeV$)	τ^- (1.7GeV)

Quantum

+2/3	Up (few MeV)	Charm (~1GeV)	Top (~170 GeV)
-1/3	Down (few MeV)	Strange(~100MeV)	Bottom (~4 GeV)

Proton=UUD
Neutron=UDD

Bosons (=Force carriers)

Photon γ m=0, q=0(EM)
Gauge bosons W^+(m=80GeV),W^- (m=80GeV),Z^0 (m=90GeV) (Weak)
Gluons g (m=, q=0) (strong)

[gravitons] (gravity)
Outside particle physics

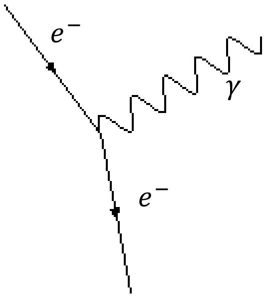
Higgs (bosons) not yet seen
q=0
m~12.5GeV

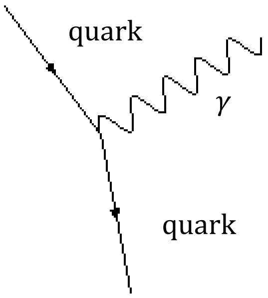
Around 1/3 of all Nobel prizes in the last 50 years have been to do with the above table
Interactions

	EM	Weak	Strong	Gravity
Neutrinos		X		X (probably)
Charged leptons	X	X		X
Quarks	X	X	X	X

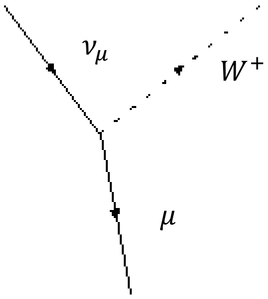
1.3 Representing interactions

EM

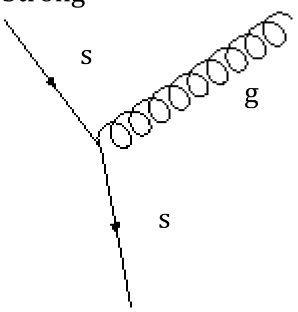




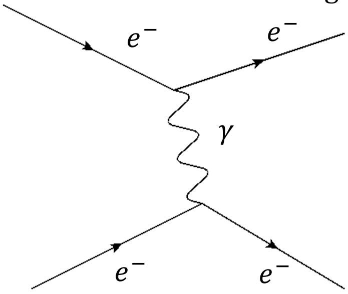
Weak



Strong



Note there is electric charge conservation i.e. total electric charge=constant $\neq$f(t)



$\rightarrow$ *time*

So forces are caused by the exchange of bosons (carrier) particles

This is the standard model of particle physics

Agrees with all observation except

- neutrinos are not massless
- Dark matter
- Dark energy
- (Gravity)
- (higgs yet to be found)

Natural units

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Kh, m, s not convenient for PP
Note c & $\hbar$ are relevant constants in eq'ns

[c]= units of $c = LT^{-1}$

$[\hbar]$ =[angular momentum]=$[MvR]$

$= ML^2T^{-1} = MV^2TET$

Lets set $c \equiv 1$ (to make eq'ns simpler)
$1a \rightarrow L = T$

i.e one length unit=1 time unit (!)

Also $\hbar \equiv 1$
$1b \rightarrow ML^2T^{-1} = 1$
But $L = T \Rightarrow ML = 1$

Ie one mass unit=one length unit=one time unit

$1c \rightarrow ET = 1$

i.e. one energy unit=$(one\ time\ unit)^{-1}$=one mass unit

Sumary

Length unit ≡ time unit

Energy unit ≡ mass unit

Reciprocals of eachother

Mpw just choose one of the few above units and set 8 of the other 3 follow (or are derived from that)
Best/convention choice is

$1\ Energy\ unit = 1eV$

$= energy\ req'd\ to\ move\ one\ electron\ thru\ 1\ volt = qV = 1.6 \times\ 10^{-19} * 1V$

$= 1.6 \times 10^{-19}J$

$1eV = 1.6 \times 10^{-19}J$
$KeV = 10^3 eV$
$M_e \sim \frac{1}{2} MeV$
Note setting $c = \hbar = 1$ simplifies eq'n e.g. reletavistic energy-momentum equ'n
SI

$E^2 = k^2c^2 + m^2c^4 \rightarrow^{k=0} E = mc^2 \rightarrow^{c=\hbar=1} E = m$

$\rightarrow^{c=\hbar=1} E^2 = k^2 + m^2 \rightarrow^{k=0} E = m$

Relationship between SI and NU

We have

$L = T = E^{-1}$

$M = E$

So a quantity Ω with SO units

$[\Omega_{SI}] = M^p L^q T^h$

Has NU

$[\Omega_{NU}] = E^p E^{-q} E^{-r} = E^{p-q-r}$

	SI	-	-	Natural units
Quantity	$[m] \rightarrow p$	$[L] \rightarrow q$	$[T]r$	$n = p - q - r$
Mass	1	0	0	$1[m] = [E], m_{proton} \sim 938MeV$
Length	*0*	*1*	*0*	*-1*
Time	*0*	*0*	*1*	*-1*
Energy=$\frac{1}{2}mv^2$	*1*	*2*	*-2*	*1*
Momentum	*1*	*1*	*-1*	*1*
Velocity	*0*	*1*	*-1*	*0*

Force= ma	*1*	*1*	*-2*	*2*
Angular mom = mvr	*1*	*2*	*-1*	*0*

How to rewrite SI units into Natural units?

$A\ distance\ a = 0.1\ fm$

$= 0.1 \times 10^{-15} m$

What is "a" in N.U.>

$a_{SI} = 0.1 \times 10^{-15} m$

Write $a_{nu} = a_{si} \hbar^a c^b$

{putting back the $\hbar$& c}

$[a_{nu}] = [a_{si}][\hbar^a][c^b]$

$= L\ E^a T^a\ L^b T^{-b}$

$= E^a L^{b+1} T^{a-b}$

In NU (LHS)

$= E^{-1}$

Equate powers

$E \Rightarrow -1 = a$

$L \Rightarrow 0 = b + 1 \rightarrow b = -1$

$T \Rightarrow 0 = a + b \rightarrow a = b = -1$

So

$$a_{nm} = \frac{a_{SI}}{\hbar c}$$

Using $\boxed{\begin{aligned} \hbar &= 1 \times 10^{-14} Js \\ &= 6.6 \times 10^{-16} eVs \end{aligned}}$

$$a_{nu} = \frac{10^{-16} m}{6.6 \times 10^{-16} \times 3 \times 10^8 ms^{-1}}$$

$$= \frac{1}{20 \times 10^8 eV} = \frac{1}{2GeV}$$

How do we rewrite natural units into SI?

Parapositronium is an unstable bound state of e^- and e^+ & has a lifetime of

$$\tau_{nu} = \frac{2}{m\alpha^5}$$

$m = m_e$

$$\alpha = \frac{1}{137}$$

$[\alpha] = 0$

Dimensionless

What is τ_{si}?

Sol'n

Write

$\tau_{si} = \tau_{nu} \hbar^a c^b$

i.e.

$$\tau_{si} = \frac{2}{m\alpha^5} \hbar^a c^b$$

So

$[\tau_{si}] = [m^{-1}][\alpha^{-5}][\hbar^a][c^b]$

$[\alpha^{-5}]$ dimless

$$= M^{-1} \frac{M^a L^{2a}}{T^a} \frac{L^b}{T^b}$$

$\frac{M^a L^{2a}}{T^a}$ →haven't used $[\hbar] = ET$ here since there's an "M" already floating around

$= M^{a-1} L^{2a+b} T^{-a-b}$

$= T$

Equate powers

M	a-1=0	$\rightarrow a = 1$
L	2a+b=0	$\rightarrow b = -2$

T	-a-b=1	$\rightarrow b = -2$

So

$$\tau_{si} = \tau_{nu}\frac{\hbar}{c} = \frac{2\hbar}{m\alpha^5 c^2}$$

Check

$$\left[\frac{2\hbar}{m\alpha^5 c^2}\right] = \frac{ML^2T^{-1}T^2}{ML^2}$$
$$= T$$

$[\hbar] = ET$
$[c] = LT^{-1}$
$E_{si} = m_{si}c^2$

$$[k_{si}] = [momentum] = [m_{si}v_{si}] = \left[\frac{mv^2}{v}\right] = \left[\frac{E}{v}\right]$$

Trivially

$$\begin{array}{ccccc} t_{si} & = & t_{nu} & \boxed{\hbar} \\ \uparrow & & \uparrow & \uparrow \\ T & = & E^{-1} & ET \end{array}$$

Also

$$\begin{array}{ccccc} d_{si} & = & d_{nu} & \boxed{\hbar c} \\ \uparrow & & \uparrow & \uparrow \\ L & = & E^{-1} & ET\ LT^{-1} \end{array}$$

Also

$$\begin{array}{ccccc} M_{si} & = & M_{nu} & \boxed{c^{-2}} \\ \uparrow & & \uparrow & \uparrow \\ M & = & E & c^{-2} \end{array}$$

Relativistic 4-momentum

1. Define "p"=p_μ

   $\mu = 0,1,2,3$
   $= (E, \underline{k})$
   $= 4 - vector$
   i.e has 4 cmpts
   Where
   $\underline{k}$ =3-momentum and has 3 cmpts
   $E = energy = scalar$

~~Will always use the symbol p for the 4- vector (4-momentum) , and k for the 3-vector (3-momentum)~~

Textbook [A.1] (II) Lorentz Transf'n

New frame's measurement of 4-momentum=f(old frame's measurement)
i.e. $p' = f(p)$

$$p' = p'_\mu = \begin{pmatrix} E' \\ \underline{k}' \end{pmatrix} = \begin{pmatrix} \gamma & & & -\beta\gamma \\ & 1 & & \\ & & 1 & \\ -\beta\gamma & & & \gamma \end{pmatrix} \begin{pmatrix} E \\ \\ \underline{k} \end{pmatrix}$$

$\uparrow$ *column vector* $= \uparrow \Gamma$ $\quad \uparrow p_\mu$

For frames moving along z axis relative to each other where

$$\beta = \frac{v}{c}$$
$$\gamma = \frac{1}{\sqrt{1-\beta^2}}$$

(III) Scalors

Eg

$p^2 = p_\mu g^\mu_\nu p^\nu$ ="p.p"

$$= (E, \underline{k}) \begin{pmatrix} 1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix} \begin{pmatrix} E \\ \\ \underline{k} \end{pmatrix}$$
$$= E^2 - \underline{k}\underline{k}$$
$$= E^2 - |\underline{k}|^2$$

$= E^2 - k^2$
=scalar
$\neq$ vector
Cf complex numbers
$z = a + ib$
$z^2 \equiv zz^*$
$= (a + ib)(a - ib)$
$\neq z.z$

1. Define
   $p_\mu = (E, \underline{k})$
   4 cmpts
   $\underline{k} = (k_x, k_y, k_z)$
   3 cmpts
2. Lorentz transf
   $$p'_\mu = \begin{pmatrix} E' \\ \underline{k}' \end{pmatrix} = (\quad) \begin{pmatrix} E \\ \underline{k} \end{pmatrix} \quad (5)$$
   $$= (\quad) p_\mu$$
3. Scalar
   $p^2 = p_\mu g^\mu_v p^v$
   $$= (E \quad \underline{k}) \begin{pmatrix} 1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix} \begin{pmatrix} E \\ \underline{k} \end{pmatrix} = (p')^2 \quad (6)$$

Turns out that if p_μ transforms according to eq5 when you change frames then $p^2 = (p')^2$
i.e. a scalar is frame invariant
In the case of 4-momenta
$$p^2 = E^2 - k^2 = \text{frame inver} = m^2 \quad (7)$$
Let us call this quantity m^2 =const
$E^2 = k^2 + m^2$
(n.u.)
$k = 0 \rightarrow E^2 = m^2$
$E = m$
(n.u.)
$\rightarrow E = mc^2$

Turns out we will never need to talk about/refer to speed or velocity v.
Just need p_μ&$\underline{k}$
Remember
$$p^2 = p_\mu g^\mu_v p^v = (E \quad \underline{k}) \begin{pmatrix} 1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix} \begin{pmatrix} E \\ \underline{k} \end{pmatrix} \equiv E^2 - |\underline{k}|^2$$

Relativistic kinematics

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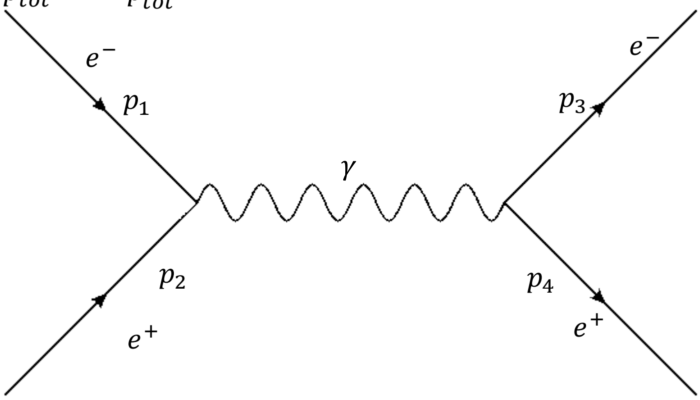
E & $\underline{k}$ are conserved for all processes

$p_{tot}^{initial} = p_{tot}^{final}$



$\Rightarrow p_1 + p_2 = p_3 + p_4$

E cmpt

$E_1 + E_2 = E_3 + E_4$

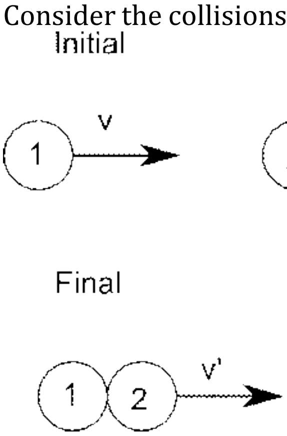
K cmpts same

<u>Classical kinematics</u>

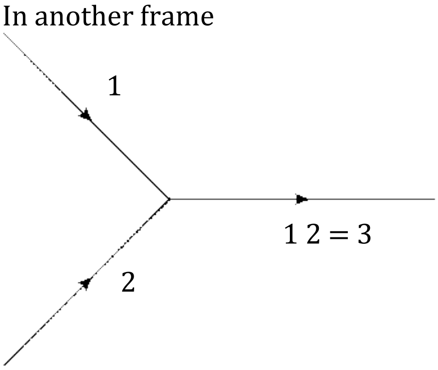
Also have E and $\underline{k}$ conserved for all processes

In case of elastic collision, kinetic energy is conserved

Consider the collisions of equal mass particles m_1 and m_2



In another frame



Classical Elastic case

i.e. Ke is conserved (not just E_{tot})

Invent a 4-cmpt classical notation

$\{ke_{tot}^{i}, \underline{k}_{tot}^{i}\} = \{ke_{tot}^{f}, \underline{k}_{tot}^{f}\}$

Curly bracket notation, which just lumps together ke and $\underline{k}$

Using $ke = \frac{1}{2}mv^2$

Using first frame

So

$$\left\{\frac{1}{2}mv^2, m\underline{v}\right\} + \{0 + \underline{0}\}$$
$$= \left\{\frac{1}{2}2mv'^2, 2m\underline{v}'\right\}$$

KE cmpt

$$\frac{1}{2}mv^2 = mv'^2 \to v' = \frac{v}{\sqrt{2}}$$

K cmpt

$$m\underline{v} = 2m\underline{v}'$$

Modulus

$$m|\underline{v}| = 2m|\underline{v}'|$$
$$|\underline{v}| = v$$

So

$$v = 2v' \to v' = \frac{v}{2}$$

Inconsistent

How can this be?

Must be that one (or more) of our initial assumptions is incorrect

$\Rightarrow ke^i_{tot} = ke^f_{tot}$

Cannot be true for this collision

i.e. if that collision happens, it is NOT elastic

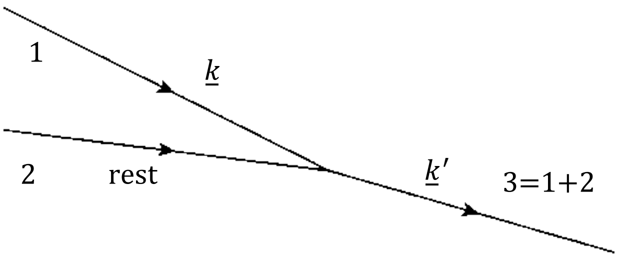
⇒ billiard balls (which are ~ elastic) won't do that

But plasticine balls will (since ke is not conserved in this case)

In inelastic collision, we have total energy conserved so some of the initial KE is transformed into another sort of energy (heat etc)

Relativistic Collision

Same collision



4-mom conservation

$$p^i_{tot} = p^f_{tot}$$

i.e.

$$\left(E^i_{tot}, \underline{k}^i_{tot}\right) = \left(E^f_{tot}, \underline{k}^f_{tot}\right)$$

i.e.

$$(K_1, \underline{k}) + (m, \underline{0}) = (E_3, \underline{k}')$$

Can only consider one spatial dimension, since

$\underline{k} = (k, 0,0)$ & $\underline{k}' = (k', 0,0)$

(both are only in x direction by conservation of momentum

$[\underline{k} + \underline{0} = \underline{k}'])$

We have

$$(E_1, k) + (m, 0) = (E_3, k')$$

$E^2 = m^2 + k^2$

$$\left(\sqrt{m^2 + k^2}, k\right) + (m, 0) = \left(\sqrt{4m^2 + k'^2}, k'\right)$$

Equate components

E: $\sqrt{m^2 + k^2} + m = \sqrt{4m^2 + k'^2}$

k: $k + 0 = k'$

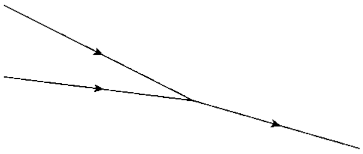
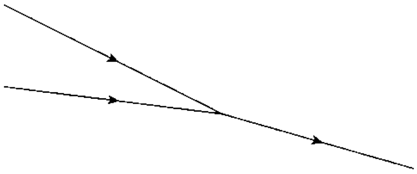
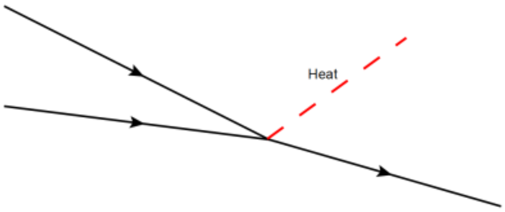
k=k'

$$\sqrt{m^2 + k^2} + m = \sqrt{4m^2 + k^2}$$

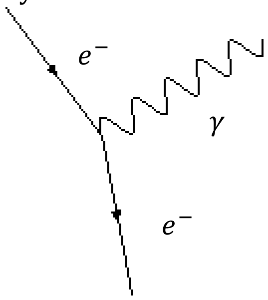
Square both sides

$$m^2 + k^2 + 2m\sqrt{m^2 + k^2} + m^2 = 4m^2 + k^2$$

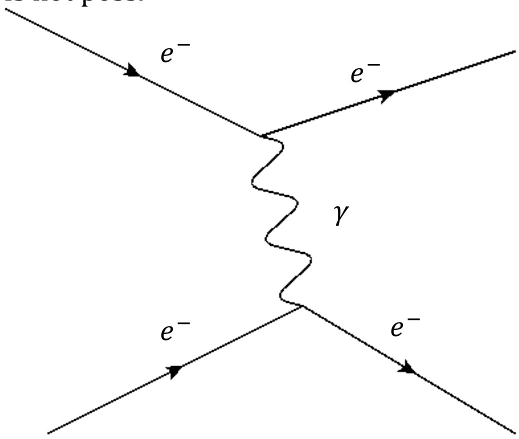
$2m\sqrt{m^2 + k^2} = 2m^2$
$\sqrt{m^2 + k^2} = m$
$\rightarrow k = 0$
So there is "no" $2 \rightarrow 1$ collision possible in relativistic mechanics

Classical	Relativistic
Elastic  Not possible	Elastic  Not possible
Inelastic 	

This says that



Is not poss.



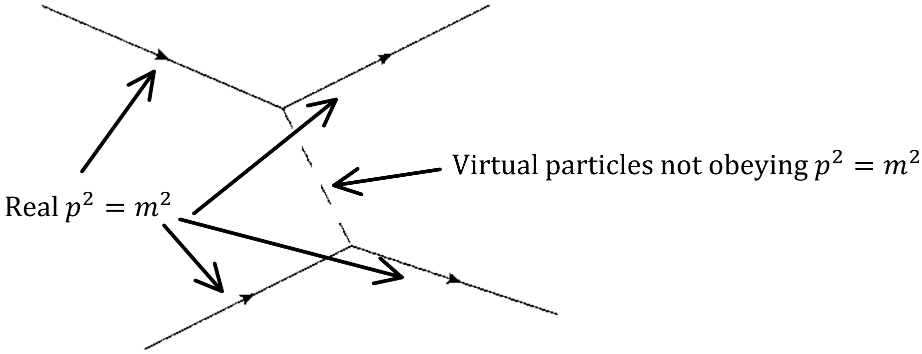
Also not poss.
So how are forces carried then?
$\Rightarrow$ if we want the above diag, one of our assumptions must be incorrect

$p_i^{tot} = p_f^{tot}$
$E^2 = m^2 + k^2$
$\uparrow$ abandon

1.3.2 Virtual Particles

Turns out that we need to abandon $p^2 = E^2 - k^2 = m^2$ (7) while maintaining $p_i^{tot} = p_f^{tot}$
Particles <u>not</u> obeying (7) are called <u>virtual</u> particles
Virtual particles "borrow" Energy from vacuum for time Δt via Heisenberg uncertainty principle

$\Delta E \Delta t \geq \hbar = 1 \; (n.u.)$



Virtual particles can only occur inside Feynman diags.
Never in external legs
$\Rightarrow$ can never be detected
How do we calculate virtual particle contributions?
Need to sum over all possible $(E, \underline{k})$ values- i.e. it's QM!
In PH-229, we only calculate particles on external legs

1.4.2 Range of Forces I

Now consider case of virtual particle (massive)
To create this particle, require $\Delta E \sim m$ energy
(since $E = mc^2 = m$ in n.u. for $\underline{k} = \underline{0}$)
$E^2 = m^2 + k^2$
This can be done for time Δt where $\Delta E \Delta t \geq \hbar = 1$
So $\Delta t \sim \frac{1}{\Delta E} = \frac{1}{m}$ (n.u.)
How far can this particle go?
Using $d = v\Delta t$ & $v = c = 1$
$\rightarrow Range = R_{nu} \sim \frac{1}{m_{nu}}$
Convert to S.I.
$R_{nu} = 1/m_{nu}$
$R_{si} = \frac{\hbar^a c^b}{m_{si} c^2}$
$[R_{si}] = \frac{[\hbar^a][c^b]}{[m_{si} c^2]}$
$L = \frac{E^a T^a L^b T^{-b}}{E}$
$a = 1, b = 1$
$\Rightarrow L = L$
$R_{si} = \hbar c / m_{si} c^2$
$= \frac{\hbar}{m_{si} c}$

m_{nu}	R_{si}
1eV	$2 \times 10^{-7} m$
1KeV	$2 \times 10^{-10} m$
1MeV	$2 \times 10^{-13} m$
$M_\pi \sim 150\ MeV$	$\sim 1\ fm = 10^{-15} m$
1GeV	$2 \times 10^{-16} m$
$M_{w,z}$	$2 \times 10^{-18} m$

Recap

$E^2 = m^2 + k^2$
$\rightarrow E = \pm\sqrt{\square}$
$\Rightarrow$Antiparticles

Summary

	Real	Virtual

	Obeys $E^2 = m^2 + k^2$	$E^2 \neq m^2 + k^2$
Particle e^-, p^+ "Usual" charge	Can occur as initial & final states Forward arrows in Feynman diagrams	Can't occur as I or f Forward arrow
Antiparticle Eg e^+, p^- Opp charge ≡ missing -ve energy state ≡ +ve energy state opp charge	Can occur as I & f states Backward arrow	Can't occur as I or f Backward arrow

1.4.2 Range of Forces II

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Recall uncertainty principle

$$\to R \sim \frac{\hbar}{mc}$$

(8)

Can now derive this properly from klein-gordon eq'n

(10)[si]

$$-\hbar^2 \frac{\delta^2 \phi}{\delta t^2} = -\hbar c^2 \nabla^2 \phi + m^2 c^4 \phi$$

Solve in static case, $i.e. \frac{\delta \phi}{\delta t} = 0$

$$\to \nabla^2 \phi = \frac{m^2 c^2}{\hbar^2} \phi$$

For radially symmetric case

$$\nabla^2 \phi = \frac{1}{r^2} \frac{\delta}{\delta r} \left(r^2 \frac{\delta}{\delta r} \right) \phi(r)$$

$$ie \frac{\delta \phi}{\delta \theta} = \frac{\delta \phi}{\delta "\phi"} = 4$$

Ie

$$\phi = f(r) \neq f(\theta, "\phi")$$

$$\to \frac{1}{r^2} \frac{\delta}{\delta r} \left(r^2 \frac{\delta}{\delta r} \right) \phi(r) = \frac{m^2 c^2}{\hbar^2} \phi(r)$$

$$\equiv \beta^2 \phi(r)$$

(*)

Where $\beta = \frac{mc}{\hbar}$

Solve by change of variable

$$\phi = \frac{\chi}{r}$$

(**)

$$\frac{\delta \phi}{\delta r} = \frac{\chi'}{r} - \frac{\chi}{r^2}$$

Where $\chi' = \frac{\delta}{\delta r} \chi$

$$\frac{\delta}{\delta r} \left(r^2 \frac{\delta}{\delta r} \right) \phi = \frac{\delta}{\delta r} (r\chi' - \chi)$$

$$= r\chi'' + \chi' - \chi' = r\chi''$$

$$(*) \to \frac{1}{r} \frac{d^2 \chi}{dr^2} = \beta^2 \frac{\chi}{r}$$

$$\frac{d^2 \chi}{dr^2} = \beta^2 \chi$$

$$\frac{d^2 y}{dx^2} = 2$$

$$\frac{dy}{dx} = 2x + C$$

$$y = x^2 + Cx + D$$

$$2y = 4$$

$$y = 2$$

Solution

$$\chi(r) = Ae^{-\beta r} + Be^{+\beta r}$$

But

$$\chi = \infty \; at \; r = \infty$$

⇒ must have B=0

So $\chi(r) = Ae^{-\beta r}$

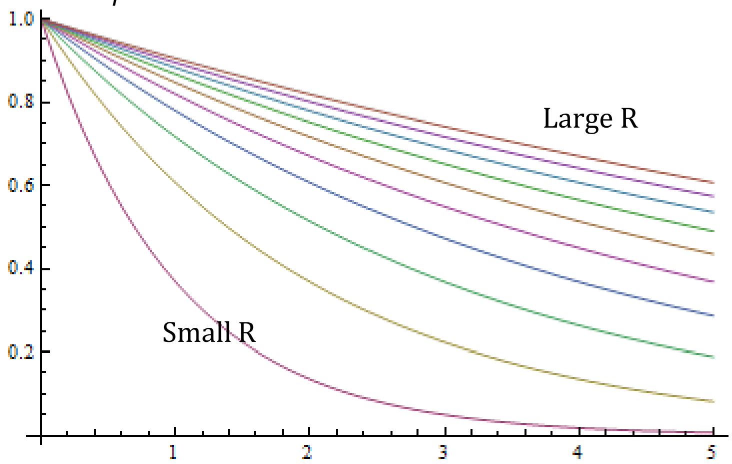
$$(**) \to \phi(r) = \frac{A}{r} e^{-\beta r}$$

$$\sim e^{-\frac{r}{R}}$$

(11)

Where

$$R = range = \frac{1}{\beta} = \frac{\hbar}{mc} \equiv (8)$$



<u>Def'n Centre of mass frame (CoM)</u>

$$\underline{k}_{tot} = \sum_i \underline{k}_i \equiv \underline{0}$$

(12)

Aka zero total 3-momentum frame

<u>Def'n</u> (invariant mass)^2 =W^2

$$\equiv (total\ 4 - mom)^2$$

$$= (p_{tot})^2$$

(13)

4-mom conservation

Ie

$$p_{tot}^i = p_{tot}^f$$

So

$$W^2 = \left(p_{tot}^i\right)^2 = \left(p_{tot}^f\right)^2$$

$$W_i^2 = W_f^2$$

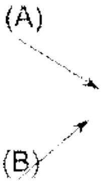
$\Rightarrow W^2$ is a conserved quantity

W^2 is also <u>frame invariant</u>

(Why? Because $W^2 = (4 - vector)^2 = scalar$)

Turns out that W^2 is a measure of how much (energy)^2 there is in a collision

Consider 2 incoming particles



$$W^2 = (p_{tot})^2 = (p_a + p_s)^2$$

$$= (E_A + E_B, \underline{k}_A + \underline{k}_B)^2$$

$$= (E_A + E_B)^2 - (\underline{k}_A + \underline{k}_B) * (\underline{k}_A + \underline{k}_B$$

$$= E_A^2 + 2E_AE_B + E_B^2 - \underline{k}_A\underline{k}_A - 2\underline{k}_A\underline{k}_B - \underline{k}_B\underline{k}_B$$

$$= E_A^2 + 2E_AE_B + E_B^2 - \underline{k}_A^2 - 2\underline{k}_A\underline{k}_B - \underline{k}_B^2$$

But

$$E_{\substack{A\\B}}^2 = m_{\substack{A\\B}}^2 + k_{\substack{A\\B}}^2$$

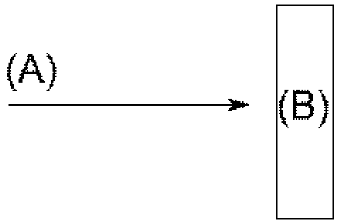
From (7)

So

$$W^2 = m_A^2 + m_B^2 + 2E_AE_B - 2\underline{k}_A\underline{k}_B$$

(14)

Case 1 "Fixed Target"



$\underline{k}_B = 0$

$(7) \rightarrow E_B = m_B$

$(14) \rightarrow W_i^2 = m_A^2 + m_B^2 + 2E_A m_B + 0 \ (*)$

Suppose we want to create a new particle C of mass m_c from above collision

We want to do this with min amount of energy W_A posible

Cheapest way is to produce C at rest, i.e. $\underline{k}_c = \underline{0}$

$\rightarrow (7) \rightarrow E_c = m_c$

Know $W_f^2 = W_i^2$

$(*) \rightarrow m_c^2 = m_A^2 + m_B^2 + 2m_B E_A^{min}$

$\rightarrow E_A^{\min} = \dfrac{m_c^2 - m_A^2 - m_B^2}{2m_B}$

Suppose $m_c \gg m_{A,B}$

So

$$E_A^{min} \sim \frac{m_c^2}{2m_B}$$

$$i.e. m_c \sim \sqrt{2m_B E_A^{min}}$$

(15)

Case 2: Colliding beams

$\underline{k}_A \neq \underline{0} \& \underline{k}_B \neq \underline{0}$

Again we want to create particle c with at least an energy $E_{A,B}$ as possible.

This is done (as above) by creating c at rest, i.e. $\underline{k}_c = \underline{0}$

$(7) \rightarrow E_c = m_c$

$(14) \rightarrow W\_$

Problem Sheet 1

27 February 2012
10:12

Q1

A)

$$R_{SI} = \frac{\hbar}{m_{SI} c}$$

$$\hbar = c = 1$$

$$R_{nu} = \frac{1}{m_{nu}}$$

B)

Want $R_{SI} = f(m_{nu})$

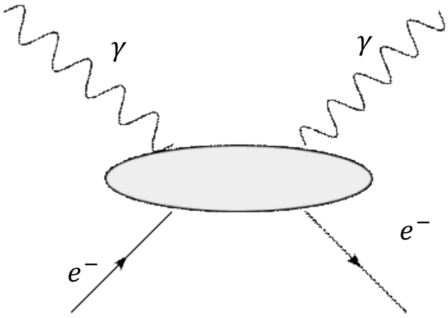
$$m_{SI} = m_{nu} \hbar^a c^b$$

$$[m_{SI}] = [m_{nu}][\hbar^a c^b]$$

Compton Scattering

29 February 2012
12:08

Arthur Compton, Washington Univ, St Louis, Nobel 1927
Consider photon interacting with e^-
Lab frame



move to more convenient frame where initial e^- is at rest

$\gamma + e^- \rightarrow \gamma + e^-$
4 mom conservation
$p_\gamma + p_e = p'_\gamma + p_e$
4 cmpt vector eqn
Ie

$$\left(E_\gamma, \underline{k}_\gamma\right) + \left(E_e, \underline{0}\right) = \left(E'_\gamma, \underline{k}_\gamma\right) + \left(E'_e, \underline{k}'_e\right)$$

$$E^2 = k^2 + m^2$$

$$\left(k_\gamma, \underline{k}_\gamma\right) + \left(m_e, \underline{0}\right) = \left(k'_\gamma, \underline{k}_\gamma\right) + \left(\sqrt{m_e^2 + k_e'^2}, \underline{k}'_e\right)$$

Note $\underline{k}$ =3 cmpt VECTOR
$k = |\underline{k}| = real\ number$
E cmpt

$$k_\gamma + m_e = k'_\gamma + \sqrt{m_e^2 + k_e'^2}$$

Sq

$$\sqrt{k_\gamma - k'_\gamma + m_e} = m_e^2 + k_e'^2$$

$$k_\gamma^2 + k_\gamma'^2 + m_e^2 + 2m_e\left(k_\gamma - k'_\gamma\right) - 2k_\gamma k'_\gamma = m_e^2 + k_e'^2$$

$$k_\gamma^2 + k_\gamma'^2 + 2m_e\left(k_\gamma - k'_\gamma\right) - 2k_\gamma k'_\gamma = k_e'^2$$

(*)

$\underline{k}$ cmpt

$$\underline{k}_\gamma = \underline{k}'_\gamma + \underline{k}'_e$$

3 cmpt equation

$$\rightarrow \underline{k}_\gamma - \underline{k}'_\gamma = \underline{k}'_e$$

Self dot product

$$\left(\underline{k}_\gamma - \underline{k}'_\gamma\right) * \left(\underline{k}_\gamma - \underline{k}'_\gamma\right) = \underline{k}'_e * \underline{k}'_e$$

$$\underline{k}_\gamma * \underline{k}_\gamma + \underline{k}'_\gamma * \underline{k}'_\gamma - 2\underline{k}_\gamma \underline{k}'_\gamma = k_e'^2$$

$$k_\gamma^2 + k_\gamma'^2 - 2k_\gamma k'_\gamma \cos\theta = k_e'^2$$

(**)

Equate LHS of (*) and (*)

$$2m_e\left(k_\gamma - k'_\gamma\right) - 2k_\gamma k'_\gamma = -2k_\gamma k'_\gamma \cos\theta$$

$$m_e\left(k_\gamma - k'_\gamma\right) = k_\gamma k'_\gamma(1 - \cos\theta)$$

$$\frac{k_\gamma - k'_\gamma}{k_\gamma k'_\gamma} = \frac{1}{m_e}(1 - \cos\theta)$$

$$\frac{1}{k'_\gamma} - \frac{1}{k_\gamma} = \frac{1}{m_e}(1 - \cos\theta)$$

But for photons

$E_\gamma = k_\gamma = \hbar\omega$

$= \omega$

(in NU)

So

$$\frac{1}{\omega'} - \frac{1}{\omega} = \frac{1}{m_e}(1 - \cos\theta)$$

Convert to SI using

$\omega = 2\pi f$

$c = f\lambda$

$\hbar = \frac{h}{2\pi}$

$\rightarrow \lambda' - \lambda = \frac{h}{m_e c}(1 - \cos\theta)$

$\neq f(\lambda)$

$= f(\theta\ only)$

Confirms particle nature of light

i.e.

$\max \Delta\lambda = \lambda' - \lambda$

Occur at $\theta = \pi$ i.e. backscattering

But double slit says light is waves

$\Rightarrow$ wave particle duality

Another example

A 10GeV e^- scatters with a p at rest. What is the momentum of the scattered p assuming all outgoing & incoming mom are linear

e^- mass insignificant compared to energy/momentum

$E_e^2 = (10GeV)^2 e^{\mp p^+} \rightarrow e^- + p^+$

$(10GeV, 1\underline{0}GeV) + (M_p, \underline{0})$

$$= \left(\sqrt{M_e^2 + k_e^2}, \underline{k}_e\right) + \left(\sqrt{m_p^2 + k_p^2}, \underline{k}_p\right)$$

But everything is colinear.

Let x axis be aligned with $\underline{k}$

$$(10,10) + (M_p, 0) = \left(\sqrt{M_e^2 + k_e^2}, k_e\right) + \left(\sqrt{M_p^2 + k_p^2}, k_p\right)$$

E cmpt

$$10 + M_p = \sqrt{M_e^2 + k_e^2} + \sqrt{M_p^2 + k_p^2}$$

K cmpt

$10 = k_e + k_p \rightarrow k_e = 10 - k_p$

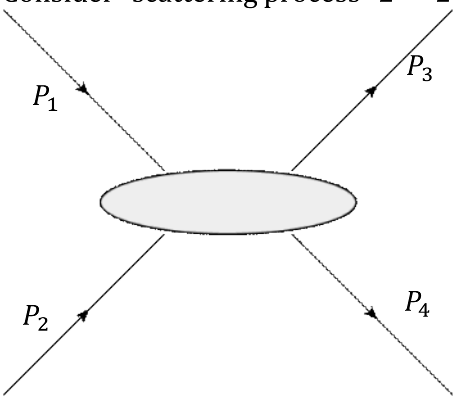
$$10 + M_p = \sqrt{M_e^2 + (10 - k_p)^2} + \sqrt{M_p^2 + k_p^2}$$

Solve

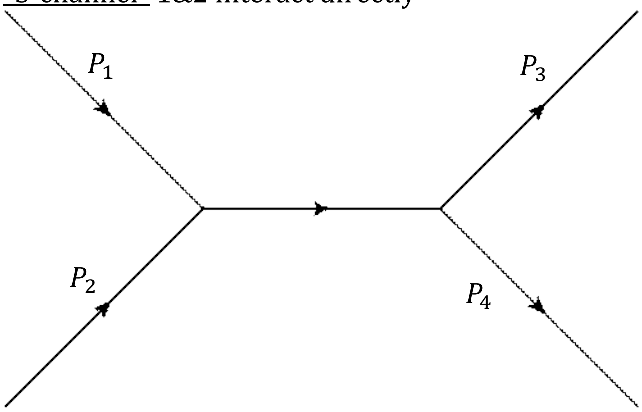
Mandelstam variables

05 March 2012
10:09

(stanley mandelstam)
Consider "scattering process" $2 \to 2$

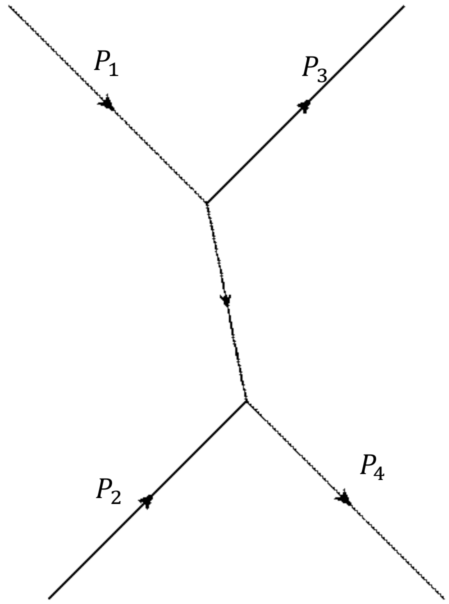


"S-channel" 1&2 interact directly



$s = (P_1 + P_2)^2$
Note at the ~~~~

"T-channel" 1&3 interact directly



"U-channel" 1&4 interact

2 The Forces of Nature

05 March 2012
10:18

Recall:
<u>Fermions</u>

$$Q=+\frac{2}{3} \quad \begin{pmatrix}u\\d\end{pmatrix}\begin{pmatrix}c\\s\end{pmatrix}\begin{pmatrix}t\\b\end{pmatrix} \text{ Quarks}$$
$$Q=-\frac{1}{3}$$

$$Q=-1 \quad \begin{pmatrix}e^-\\\nu_e\end{pmatrix}\begin{pmatrix}\mu^-\\\nu_\mu\end{pmatrix}\begin{pmatrix}\tau^-\\\nu_\tau\end{pmatrix} \text{ leptons}$$
$$Q=0$$

<u>Bosons</u>

γ photon-EM
g gluon- Strong
$W^\pm, Z^0$ gauge bosons- Weak
(H Higgs)

<u>Strength of Forces</u>

Expt shows typical lifetimes of particles which decay via these 3 forces/interactions

Interaction	Typical Lifetimes τ
Strong	$10^{-23}\ s$ ~time taken for light to cross a proton
EM	$10^{-16} \to 10^{-21} s$
Weak	$10^{-7} \to 10^{-13} s$

There are exceptions: e.g. neutron

$n \to p + e^- + \bar{\nu}_e$

$t \sim 10^3 s$

Since $\tau = \text{lifetime} \propto \frac{1}{\text{Interaction Strength}}$

$\Rightarrow$ we can get an idea of the strength from τ

<u>EM interactions</u>

γ couples with/interacts with any particle with non-zero electric charge
It does <u>not</u> change the particle species
⇒it does not change the electric charge
⇒Q is conserved quantity
EM interactions are responsible for all atomic & molecular physics, aka chemistry
Recall the Range of forces II, Klein-Gordon eq'n was solved

$$(11) \to \phi(r) \sim \frac{1}{r} e^{-\beta r}$$

Where

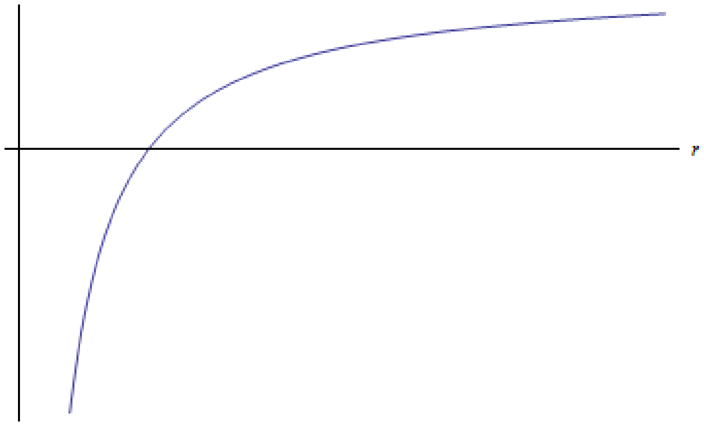
$$\beta = \frac{mc}{\hbar}$$

But for photons, $m_\gamma = 0$

So $\beta = 0$

$$\phi(r) \sim \frac{1}{r}$$

i.e. usual electrostatic potential



⇒ this is possibility of bound and unbound solutions

e.g.

H atom, e^- is bound

H^+ ion, e^- is unbound

Since there are two types of electric charge, +ve and -ve, the el. Charge can be represented on a (single) number line

Strength of EM interactions $\sim e^2$

In detail

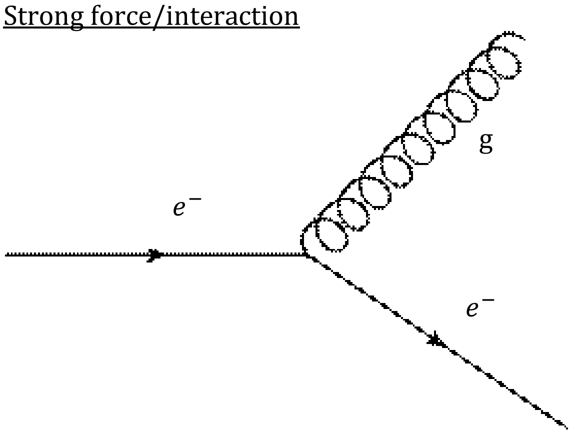
$$\alpha \equiv \frac{e^2}{4\pi\hbar c\epsilon_0}$$

$$= \frac{1}{137.035999070(98)}$$

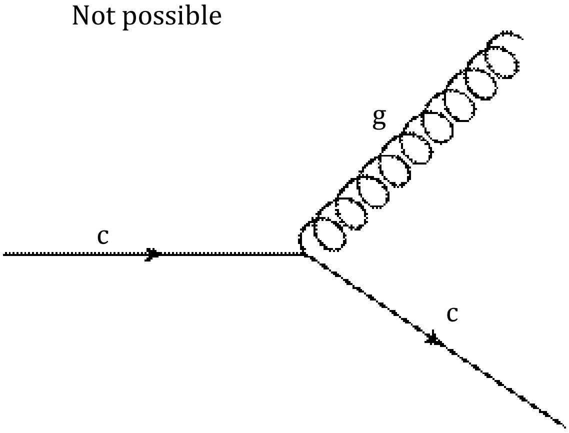
=Fine structure constant

10 Sig figs!

Strong force/interaction



Not possible



Is possible

It does not change particle species, but it does change colour charge of the quark

Why? G carry colour charge

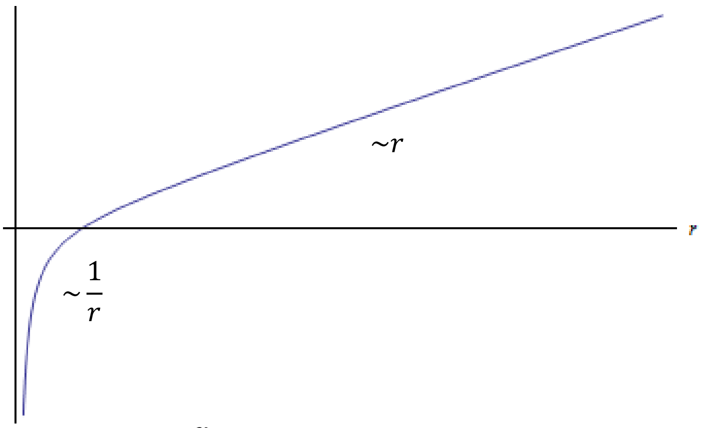
Rule 1:

g interact with anything with colour charge

Rule 2:

Colour charge is conserved

Strong interaction is responsible for binding of quarks inside protons & neutrons (and other hadrons)



$V(r) = -\frac{\alpha}{r} + \sigma r$

⇒bound states only!

$\sqrt{\sigma} = 440 MeV$

$\rightarrow Force = -\frac{\delta V}{\delta r} = 15\ tonnes * g = \sigma$

Quarks can never* become unbound!

Almost true.

Turns out that at temperatures of $10^{12} k$ and/or density of 10^9 tonnes/cm^3 the stong potential bends down

→ possibility of free quarks!

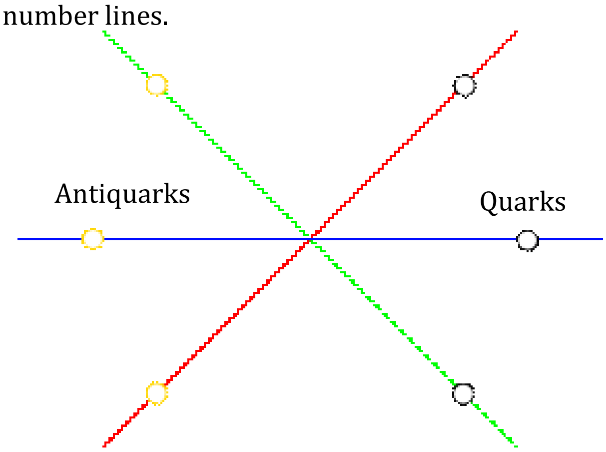
"Quark-Gluon plasma"

First μsec after big bang

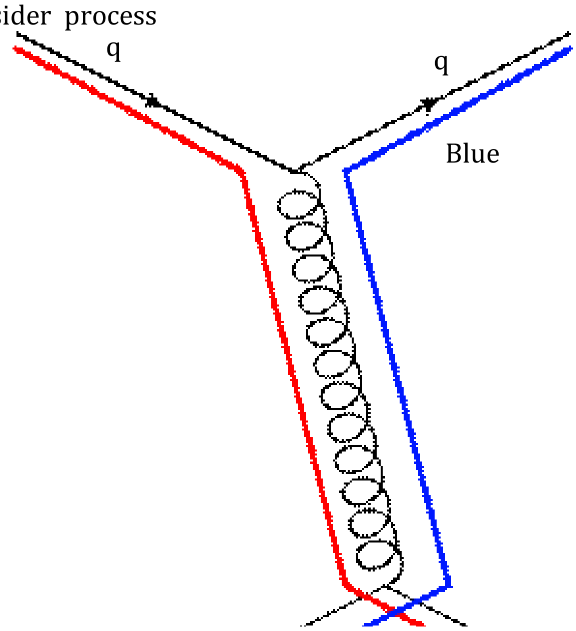
Cores of some neutron stars (?)

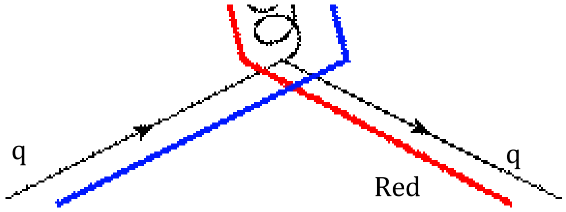
LHC when colliding Pb ions

The quantity corresponding to electric charge in the strong interaction is "colour charge". This can be represented as 3 number lines.

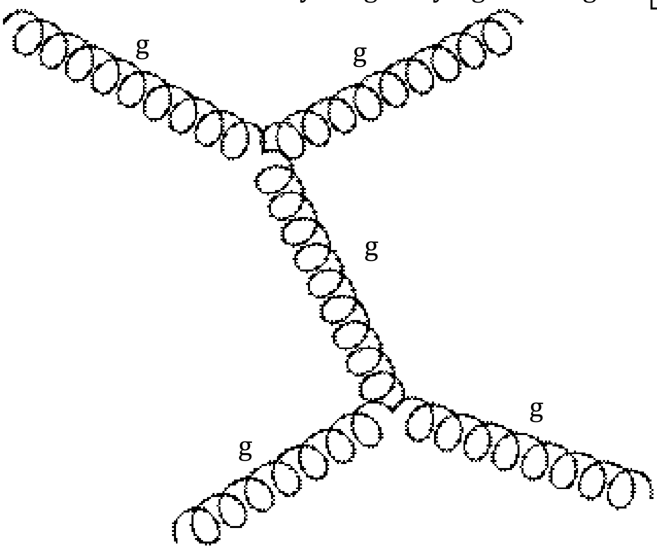


Consider process

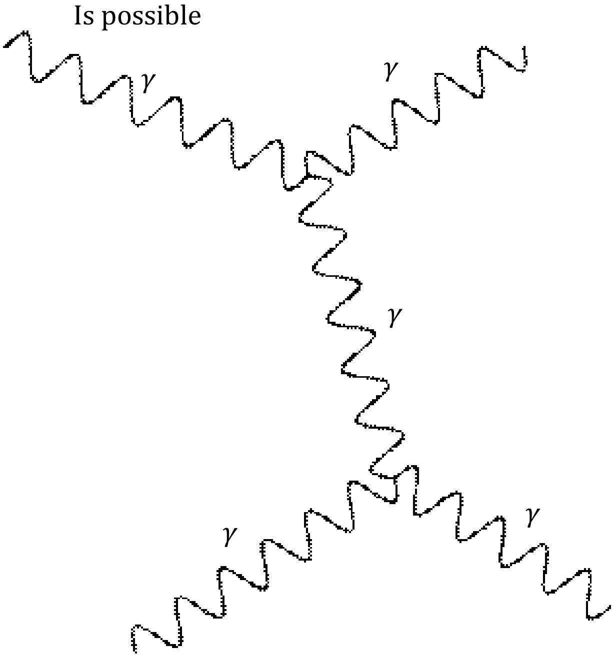




i.e the gluon in this case is blue-antired
⇒ gluons contain colour
How many colour combinations can gluons have
$\Rightarrow 3 * 2 = 6$
Turns out there are also 2 "diagonal" combinations → 8 in total
Contrast this with EM case. Photons do NOT carry el. Charge!
Note: gluons interact with any particle which carries colour ⇒ Gluons self-interact
Photons interact with anything carrying el. Charge ⇒ Photons do NOT self-interact



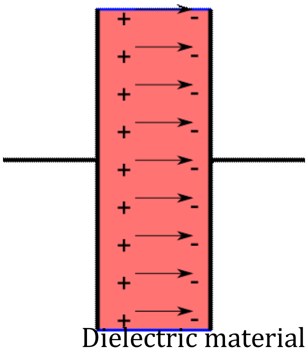
Is possible



Not possible

7.1.2 Vacuum polarisation

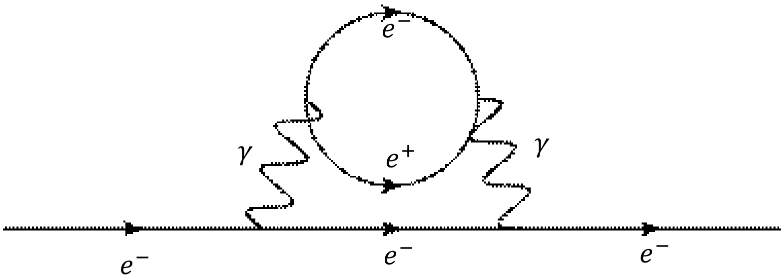
Recall Dielectric effect (capacitor in dielectric material



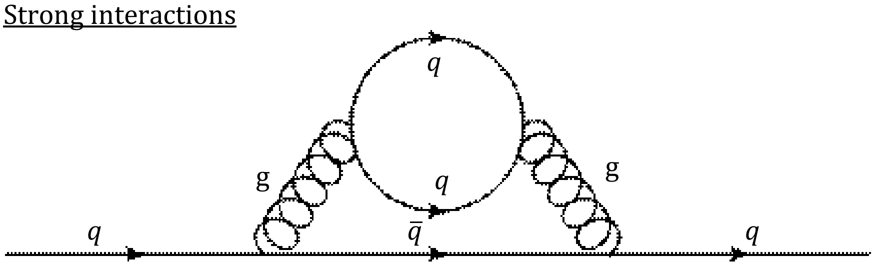
Dielectric material (i.e. contains "free charges")

→ reduction in effective charge on each plate
∃ analogous effect in EM

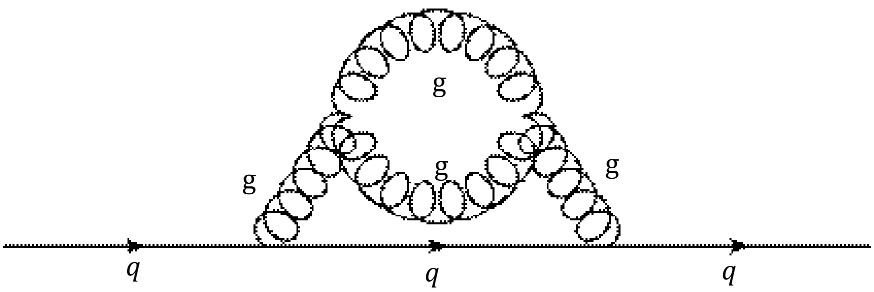
Reduction in effective charge of the incoming e^- due to e^+ "popping out of the vac"



<u>Strong interactions</u>



I: Reduction



II: Enhancement

Type 2 doesn't occur in EM
Overall, I+II= net effect is anti-screening (enhancement)

Force decreases as distance decreases- "asymptotic freedom"
How can $v(r) \sim \sigma r \text{ as } r \to \infty$?
Consider EM and $v_{em}(r) \sim \frac{1}{r}$
In strong interaction, flux lines are

Defn: Hadron

Any bound state of quarks (or antiquarks)
Turns out there are 2 ways of doing this to form colourless states

1. Baryons

qqq (3 quarks)
e.g. $p = uud, n = udd$
$\Lambda^+ = suu$

2. Mesons

$q\bar{q}$ (quark + antiquark)
e.g. $\pi = u\bar{u} + d\bar{d}$
$k^+ = \bar{s}u$

2.2.2 Quantum numbers

08 March 2012
12:30

$\binom{u}{d}\binom{c}{s}\binom{t}{b}$

Since the strong interaction doesn't change quark species

→ conserved quantum number

$= N_u, N_d, N_c, N_s, N_t, N_b$

Where

$N_u =$ number of u quarks $-$ number of $\bar{u}$ quarks etc

For historic reasons, N_u, N_d are not often used, but 2 other quantities take their place

Defn: El. Charge

$$= Q = \frac{2}{3}(N_u + N_c + N_t) - \frac{1}{3}(N_d + N_s + N_b)$$

(18)

Defn: Baryon number

$$= B = \frac{1}{3}(N_u + N_d + N_c + N_s + N_t + N_b)$$

(19)

= counts # of baryons

Proton, B=1

Meson, B=0

For completeness, historically, the following terms are used

"strangeness"$= S = -N_s$

"charmness"$= C = +N_c$

"bottomness"$= \tilde{B} = -N_b$

"Topness"$= T = +N_t$

Above quantum numbers can be used to categorise interactions.

Recall strong interaction <u>conserves</u> $N_{u,d,\ldots,t,b}$

$d \to u + \nu_e$

Breaks Q: $-\frac{1}{3} \to +\frac{2}{3}$

Breaks $N_{u,d}$

⇒ can't be strong!

$K^+ \to \pi^0 + \mu^+ + \nu_\mu$

i.e.

$u\bar{s} \to u\bar{u} + \cdots$

N_u $1 \to 1 - 1 + 0$

Broken

N_d $0 \to 0$

Ok

N_s $-1 \to 0$

Broken

Q $+1 \to 0 + 1 + 0$

Ok

$K^0 \to K^+ + \pi^-$

$d\bar{s} \to u\bar{s} + \bar{u}d$

i.e. meson contains $q_1\bar{q}_2$

K=kaon contains either an s or $\bar{s}$ and a light quark/antiquark i.e. $u, \bar{u}, d, \bar{d}$

N_U: $0 \to 1 - 1$

Yes

N_d $1 \to 1$

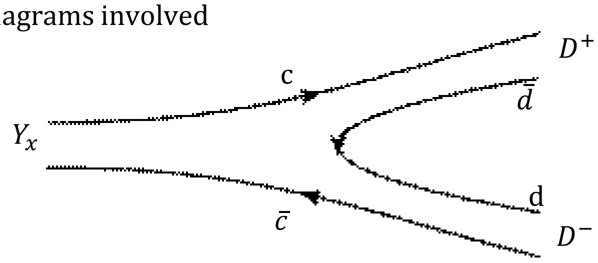
Yes

N_s $-1 \to -1$

Yes

$Q: 0 \rightarrow 1 - 1$
$B: 0 \rightarrow 0$
$\Rightarrow$ strong interaction

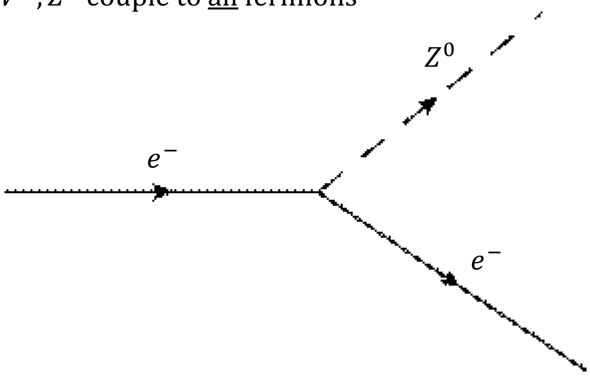
How do we do calculations involving the strong force?
Diagrams involved

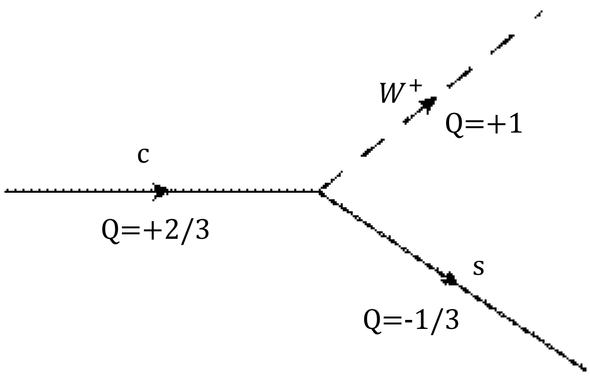


V. large number of gluon interactions

Weak interaction

W^+, W^-, Z^0 couple to all fermions





$W^\pm$ changes electric charge of fermion by ∓ 1
Z^0 doesn't change el. Charge
Note: weak interactions can change fermion species!
Recall (11) (range of forces)
$\rightarrow \phi(r) \sim \frac{1}{r} e^{-\frac{r}{R}}$
Where

$$R = \frac{\hbar}{mc}$$

Here, $m = M_{Z^0}$ or $M_{W^\pm}$
$M_{Z^0}, M_{W^\pm} \sim 100 GeV$
$\rightarrow R \sim 2 \times 10^{-3} fm$
Proton diameter~1fm
i.e. virtually point-like
$Z^0, W^\pm$ only discernible in recent (>1983) expt, when energy of beams is high enough

Archetypal weak (radioactive) decay
Nuclei $(Z, A) \rightarrow (Z+1, A) + e^- + \overline{\nu_e}$
Z= # protons
A=#p+#neutrons
1890's
Which is really

Nucleon (aka, protons & neutrons)

$n \to p + e^- + \bar{\nu}_e$

Which is really

Quarks. $d \to u + e^- + \bar{\nu}_e$
Which
Which is really
Fundamental $W^{\pm}, Z^0$
$d \to u + W^- \to u + e^- + \bar{\nu}_e$

Interesting facts about neutrinos
A ν can travel through 6 trillion tonnes of Pb w/o interacting
Big Bang produced lots of ν.
On average, the universe filled with 339 of these ν per cm^3
2×10^{21} ν pass through your body in your lifetime
Chances of these interacting with you is 1 in 4

2.1 Lepton Quantum Numbers

14 March 2012
12:02

Recall $N_q = \# q \ - \ \#\bar{q}$
Analogous, but generalised quantity for leptons
$L_e = \# \, e^- - \#e^+ + \#\nu_e - \#\bar{\nu}_e$
Similar for L_μ & L_τ
So

$L_e = "N_e" - "N_{\nu_e}"$

In earlier notation

e.g. for e^-: $L_e = +1$

$\bar{\nu}_\mu$: $L_e = 0; L_\mu = -1$

The (minimal) standard model of particle physics (i.e. EM +Weak+Strong with all neutrinos massless) says L_e&L_μ&L_τ are separately conserved
Note, since expt shows that $M_\nu \neq 0, L_{e,\mu,\tau}$ are actually not exactly conserved (!)
Pictorially,

$\begin{pmatrix} e^- \\ \nu_e \end{pmatrix} \begin{pmatrix} \mu^- \\ \nu_\mu \end{pmatrix} \begin{pmatrix} \tau^- \\ \nu_\tau \end{pmatrix}$

Weak interactions only allowed vertically

8.2.1

e.g. $\mu^- \rightarrow e^- + \nu_\mu + \bar{\nu}_e$

L_e; $0 \rightarrow 1 + 0 - 1$

Ok

L_μ; $1 \rightarrow 0 + 1 + 0$

Ok

e.g. $\nu_\mu \rightarrow e^- + \tau^- + \bar{\nu}_\tau$

L_e; $0 \rightarrow 1 + 0 + 0$

No

L_μ; $1 \rightarrow 0 + 0 + 0$

No

L_τ; $0 \rightarrow 0 + 1 - 1$

No

Forbidden in (min) standard model

8.2.3 Weak interaction of Quarks
None of $N_{u,d,c,\ldots}$ are conserved individually
Pictorially

$\begin{pmatrix} u \\ d \end{pmatrix} \begin{pmatrix} c \\ s \end{pmatrix} \begin{pmatrix} t \\ b \end{pmatrix}$

Vertical and diagonal W^+ allowed, but not horizontal

Cabibbo Angle (1963)

2 generations only

We've seen weak interactions of leptons occur only "vertically", but weak interactions of quarks occur both vertically & diagonally. To make sense of this introduces a "change of variables" $\rightarrow d'$ & s'

Def'n

$d' = d \cos\theta_c + s \sin\theta_c$

$s' = -d \sin\theta_c + s \cos\theta_c$

(20)

$\begin{pmatrix} d' \\ s' \end{pmatrix} = \begin{pmatrix} & R_\theta & \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix}$

"weak basis"=Roth Max thru θ * Orig "mass" basis

θ_c =cabibbo angle $\approx 13°$

In the new, rotated basis, weak interactions occur only vertically, i.e.

$\begin{pmatrix} u \\ d' \end{pmatrix} \begin{pmatrix} c \\ s' \end{pmatrix}$

Analogous to leptons, only vertical allowed

"quark-lepton" symmetry

Using (20)$\rightarrow$

$\binom{u}{d\cos\theta_c + s\sin\theta_c}\binom{c}{-d\sin\theta_c + s\cos\theta_c}$

Note

$\cos\theta_c = 0.97 \approx 1$

$\sin\theta_c = 0.27 \approx 0$

$u \to d$ much more likely than $u \to s$

So transition $u \to d + W^+$

Has a factor $\cos\theta_c$; "Cabibbo Allowed"

$u \to s + W^+$

Has a factor of $\sin\theta_c$; "Cabibbo Suppressed"

The above interactions change the charge of the quark, so they emit a $W^\pm$

→ called "charged currents"

Those that emit a Z^0 do <u>not</u> change quark's charge

→ called neutral currents

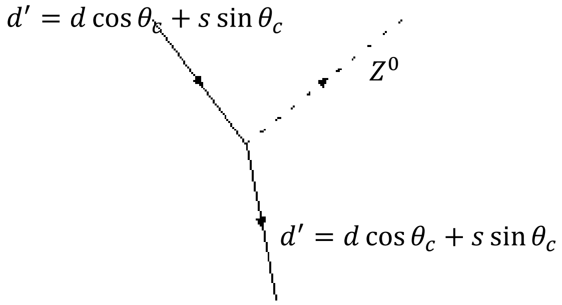
e.g. $u \to u + Z^0$

Q: is $c \to u + Z^0$ possible?

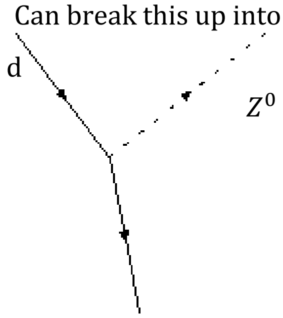
i.e. flavour changing neutral currents

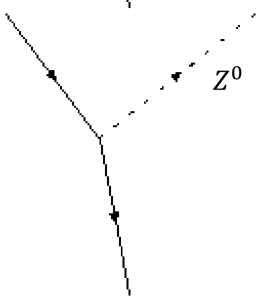
Can study this as follows

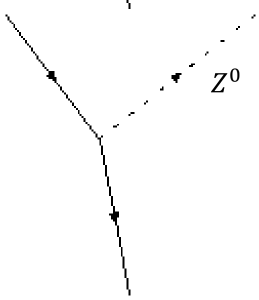
$d' \to d' + Z^0$

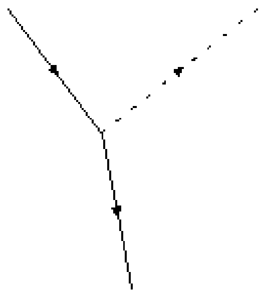


Can break this up into 4 diagrams









Elastic e^{-}+p scattering

26 March 2012
10:07

i.e.

$$e^- + p \to e^- + p$$

$$\frac{ds}{d\Omega} = \left(\frac{ds}{d\Omega}\right)_R G_E(q^2)$$
(23)

If p recoils, (23) generalises to

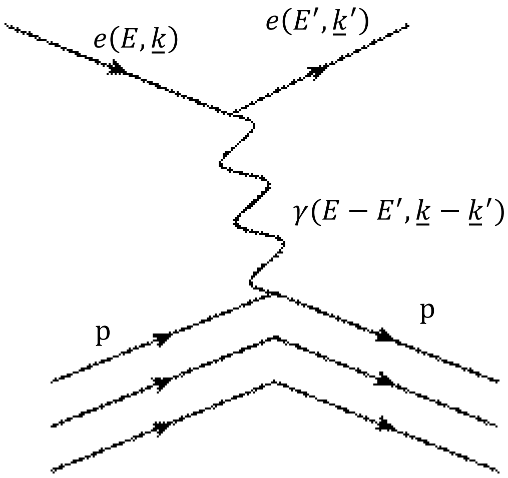
$$\left(\frac{ds}{d\Omega}\right) = (\dots)G_E(Q^2) + (\dots)G_M(Q^2)$$
(24)

(...)=some factor

Where

$$Q^2 = (k - k')^2 - (E - E')^2$$
(25)

$= -(4 - mom)^2$ transfered from e^- $to\ p$



Expt

$\frac{ds}{d\Omega}$

Measured

Recall

$G_E(Q^2) = F.T.of\ p(r)$

(charge distribution)

Since $G_E(Q^2) \neq const$

Therefore proton is not pt-like ($i.e. p(r) \neq \delta f'n$) so proton has some structure!!

What's it made of?

7.4 Deep Inelastic Scattering

26 March 2012
10:19

See fig A.

This is clearly inelastic since $x \neq proton$

$E_\gamma \approx 10GeV$

$\gg M_p \approx 1GeV$

$\gamma's\ wavelength \ll size\ of\ proton$

Recall Q' =Lorentz scalar

$= (\underline{k} - \underline{k}')^2 - (E - E')^2$

Def'n ν:

$2\ m_\nu = W_x^2 + Q^2 - M^2$

(26)

Where

$M = M_p$

$W_x^2 =$ (Invariant mass)^2

$\rightarrow \nu = E - E'$

(not proven)

Def'n x=Bjorken variable

$\frac{Q^2}{2M_\nu}$

(27)

→x=fraction of p's momentum carried by struck quark

(Proven in sheet 3)

Note for inelastic scattering

$W_x = f(E'\&\theta)$

But for elastic scattering

$W_x = M_p^2$

Elastic cross section (24) generalises to

$\frac{d\sigma}{dE'd\Omega} = (\ldots)F_1(x, Q^2) + (\ldots)F_2(x, Q^2)$

$F_{1,2} =$ "structure function"

Inelastic generalisation of form factors

Expt. 1969 SLAC

$\rightarrow F_{1,2}(x, Q^2) = f(x\ only)$

For $Q^2 \gg M$

"Bjorken Scaling"

Since $F_{1,2}(Q^2) \approx const$, it's F.T. (≈ charge distribution) is a δ function

⇒ thing being probed/hit is a point-like particle i.e. quark is fundamental!

Further studies show that $F_1\&F_2$ from experiment are consistent with spin1/2 (fermions), el. Charge $\pm\frac{1}{3}, \pm\frac{2}{3}$

i.e. quarks

Summary

Elastic $e^- + p \rightarrow e^- + p$

Low energy

$\leq \sim 5GeV$

$G_E(Q^2) \neq constant \therefore fourier\ transfrom\ \neq \delta$ ∴ thing being probed (proton) is not point like

Inelastic $e^- + p \rightarrow e^- + hadrons\ (x)$

High energy

$5 \rightarrow 20\ GeV$

$F_2(Q^2) = const \therefore F.T. = \delta$ i.e. thing being probed (quark) is point like

Practice problem sheet 2

27 March 2012
11:07

1. $V(r) \sim \sigma r$ at large r,

   So

   $F(r) = -\frac{\delta V}{\delta r} = [E][L^{-1}] = -\sigma$

   $|\underline{F}| = \sigma$

   $F_{SI} = F_{NU} \hbar^a c^n$

   $[F_{SI}] = [F_{NU}][\hbar^a][c^b]$

   $EL^{-1} = E^2 E^a T^a L^b T^{-b}$

   $E: 1 = 2 + a \rightarrow a = -1$

   $L: -1 = b \rightarrow b = -1$

   $T: 0 = a - b$

   $F_{SI} = \frac{\sigma}{\hbar c}$

   $= \frac{440^2 MeV^2}{6.6 \times 10^{-16} eVs\ 3 \times 10^8 ms^{-1}} = \frac{440^2 (10^6)^2 eV^2}{20 \times 10^{-8} eVm} - (\square) \frac{eV}{m}$

   $= (\square) 1.6 \times 10^{-19} \frac{J}{m}$

   $= 16 \times 10^4 N$

   $= mg \rightarrow m = 1.6\ \times 10^3 kg = 16\ tonnes$

2. $E^2 = m^2 + k^2$

   $\gamma + x_{(rest)} \rightarrow x + x$

   $(k, \underline{k}) + (M, \underline{0}) = (E_1, \underline{k}_1) + (E_2 + \underline{k}_2)$

   (*)

   $M = M_x$

   $\underline{k}_1 = \underline{k}_2 = \underline{k}_x$

   (*)$\rightarrow$ $(k, \underline{k}) + (M, \underline{0}) = 2\left(\sqrt{M^2 + k_x^2}, \underline{k}_x\right)$

   $\underline{k}\ cmpt\ \ \underline{k} = 2\underline{k}_x$

   $E\ cmpt\ \ k + M = 2\sqrt{M^2 + \left(\frac{k}{2}\right)^2}$

   $Sq: k^2 + 2kM + M^2 = 4\left(M^2 + \frac{k^2}{4}\right) = 4M^2 + k^2$

   $2kM = 3M^2$

   $k = \frac{3M}{2} = k_{min}$

3. $p^2 = E^2 - k^2 = m^2$

   $s = (p_1 + p_2)^2 = p_1^2 + p_2^2 + 2p_1.p_2 = m_1^2 + m_2^2 + 2p_1.p_2$

   $t = (p_1 - p_3)^2 = p_1^2 + p_3^2 - 2p_1.p_3 = m_1^2 + m_3^2 - 2p_1.p_3$

   $u = (p_1 - p_4)^2 = p_1^2 + p_4^2 - 2p_1.p_4 = m_1^2 + m_4^2 - 2p_1.p_4$

   $s + t + u = m_1^2 + m_2^2 + m_3^2 + m_4^2 + 2m_1^2 + 2(p_1.p_2 - p_1.p_3 - p_1.p_4)$

   $p_1.p_3 = (E_1, \underline{k}_1).(E_3, \underline{k}_3)$

   $= E_1 E_3 - \underline{k}_1.\underline{k}_3$

   $= |\underline{k}_1||\underline{k}_3| \cos\theta$

   $= k_1^x k_3^x + k_1^y k_3^y + k_1^z k_3^z$

   $2m_1^2 = 2p_1^2$

   $= (\square) + 2p_1^2 + 2p_1.(p_2 - p_3 - p_4)$

   $= (\square) + 2p_1.(p_1 + p_2 - p_3 - p - 4)_{(=0)}$

   $= m_1^2 + m_2^2 + m_3^2 + m_4^2$

4. Conservation of Q numbers

   i. $\pi^- + p \rightarrow \pi^- + \pi^+ + n$

      $d\bar{u} + uud \rightarrow d\bar{u} + \bar{d}u + udd$

      $N_u = -1 + 2 \rightarrow -1 + 1 + 1$

yes

$N_d = 1 + 1 \rightarrow 1 - 1 + 2$

yes

$EM\ yes \rightarrow no$

$Strong\ \ yes \rightarrow yes$

ii. $\gamma + p \rightarrow \pi^+ + n$

$\gamma + uud \rightarrow u\bar{d} + udd$

$N_u = 2 \rightarrow 1 + 1$

Yes

$N_d = 1 \rightarrow -1 + 2$

Strong or EM

But there's a γ → can't be strong ⇒ EM

iii. $\nu_\mu + n \rightarrow \mu^- + p$

$\nu_\mu + udd \rightarrow \mu^- + uud$

$N_u = 1 \rightarrow 2$

No

$N_d = 2 \rightarrow 1$

No

$L_\mu = 1 \rightarrow 1$

Yes

⇒weak

iv. $\pi^0 \rightarrow e^- + e^+ + e^- + e^+$

$u\bar{u} \rightarrow 2e^- + 2e^+$

$N_U = 0 \rightarrow 0$

$L_e = 0 \rightarrow 2 - 2$

Involves leptons so can't be strong

$\Rightarrow EM$

v. $p + \bar{p} \rightarrow \pi^+ + \pi^- + \pi^0$

$uud + \bar{u}\bar{u}\bar{d} \rightarrow u\bar{d} + \bar{u}d + u\bar{u}$

$N_u = 2 - 2 \rightarrow 1 - 1 + 0$

$N_d = 1 - 1 \rightarrow -1 + 1$

⇒ strong

vi. $u \rightarrow d + W^+$

Weak

vii. $c \rightarrow c + \gamma$

$Q:\ +\frac{2}{3} \rightarrow +\frac{2}{3}$

Involves $\gamma \Rightarrow EM$

Summary

23 April 2012
10:03

1. The basics
2. E.M., Strong, Weak
3. Quarks, Hadrons & colour
4. Expt Methods
5. Open questions

ν masses & oscillations

Recall Cabibbo theory of quarks in weak interactions

$d' = d\cos\theta_c + s\sin\theta_c$

$s' = -d\sin\theta_c + s\cos\theta_c$

Generalising

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} CMK \\ (3\times 3\ matrix) \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

There may be analogous mixing in ν- sector

$$\begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix} = \begin{pmatrix} MNS \end{pmatrix} \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix}$$

MNS=Maki-nakagawa-sakato 3x3 matrix

i.e. ν_e is a mixture of $\nu_1, \nu_2 \& \nu_3$

Turns out that if you initially have an ν_e

$|\nu_e(t=0)\rangle = \cos\alpha\,|\nu_1\rangle + \sin\alpha\,|\nu_2\rangle$

(2 flavour case

At a later time

$|\nu_e(t\neq 0)\rangle = A'(t)\,|\nu_1\rangle + B'(t)\,|\nu_2\rangle$

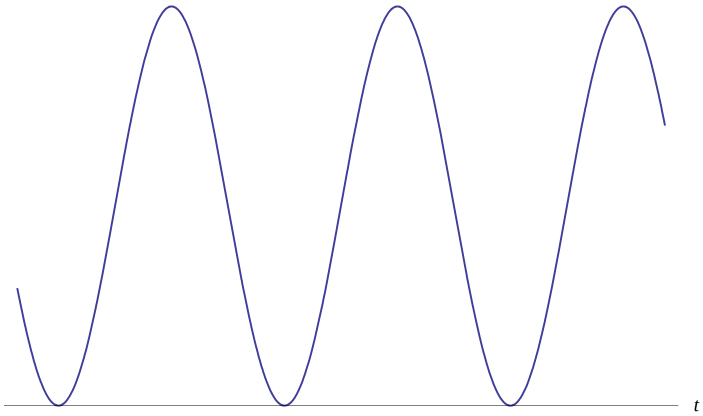
Re-write $|\nu_{1,2}\rangle$ in terms of $|\nu_{e,\mu}\rangle$

$\rightarrow |\nu_e(t\neq 0)\rangle = A(t)\,|\nu_e\rangle + B(t)\,|\nu_\mu\rangle$

Where

$A(t) = e^{-iM_1 t}\cos^2\alpha + e^{-iM_2 t}\sin^2\alpha$

$P(\nu_e \rightarrow \nu_\mu) = |A(t)|^2$



This explains the solar ν problem-

Why do we not see enough ν_e from sun?

Homestake solar ν detector (1970s)

Salsbury detector (1999+)

$p + Be^7 \rightarrow B^8 + \gamma$

$Be^8 + e^+ + \nu_e$

Tried to se ν_e, but fewer than predicted

Now ν beams are "fired" from one experiment (e.g. CERN & Fermilab) through earth & detected elsewhere:

(CERN →Gran Sasso Lab, Italy)

(Fermilab→Minnesota)

$M_{\nu_e} \sim\ \leq few\ eV$

$M_{\nu_\mu} \sim\ \leq few\ hundred\ KeV$

$M_{\nu_\tau} \sim \leq few\ MeV$

Higgs Boson

"Gives mass" to fundamental particles

Not quite observed

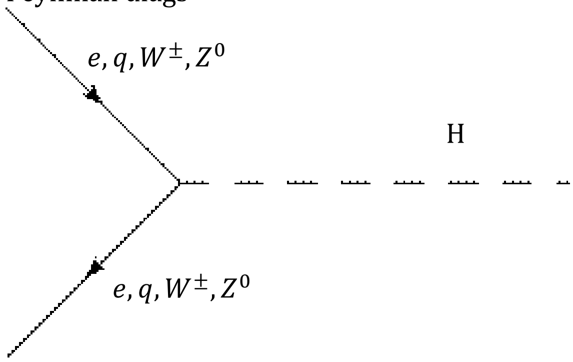
LHC (£3Bn) built for it

Current bounds $100GeV \leq M_H \leq 130GeV$

Hints that it may be

$M_H \approx 125GeV$

Feynman diags

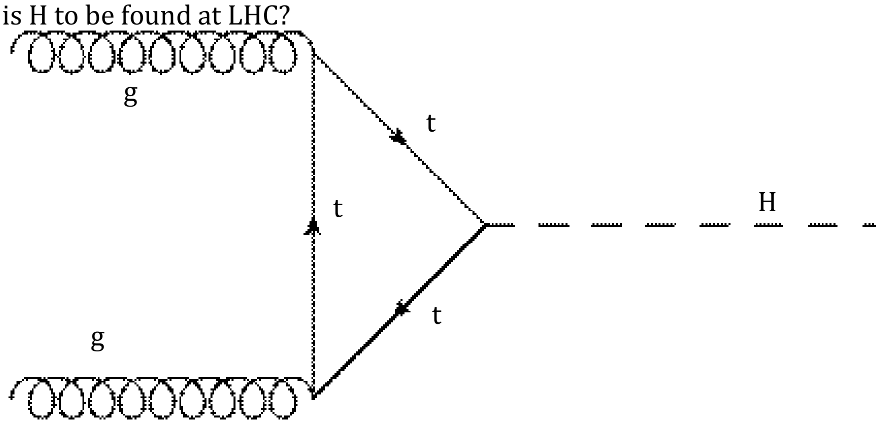


How does it "give" mass?

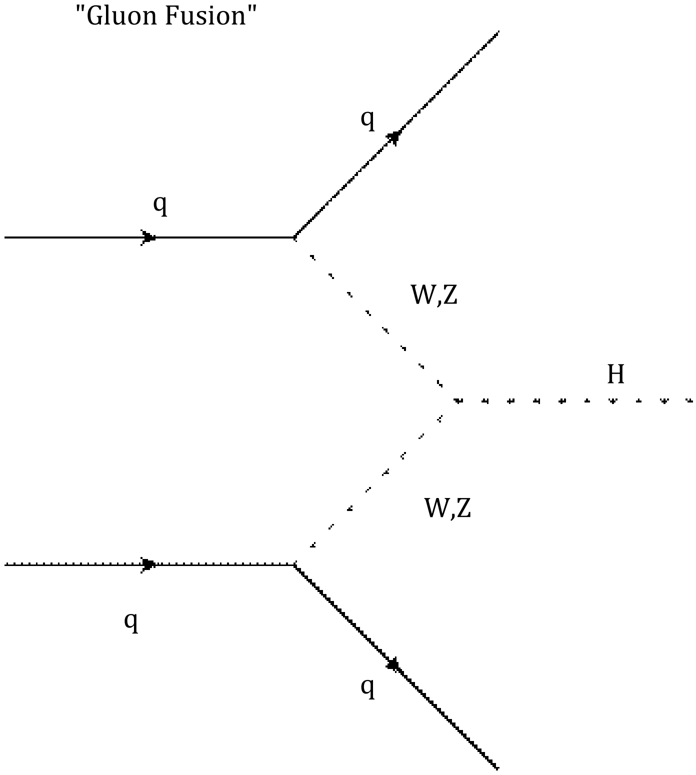
The quark "feels" the presence of H and is "slowed" down by it →q has mass.

Handwavium

How is H to be found at LHC?



"Gluon Fusion"



Matter-antimatter asymmetry

Known universe is almost entirely matter (baryons) & not anti-matter
The Std Model theory is (almost) matter-antimatter symmetric
Experiment says Universe ≈ all matter
Why? Answer- "Don't Know"

Dark matter
Stars rotate around galaxies much faster than can be accounted for by the seen/observed matter in the galaxy

Gravity $F_g - ma$
Circular Orbit, $\frac{GMm}{r^2} = \frac{mv^2}{r}$
$$v = \frac{2\pi r}{T}$$
$$\frac{GM}{r} = 4\pi^2 \frac{r^2}{T^2}$$
$$\rightarrow T^2 \propto \frac{1}{M}$$
T observed to be small i.e. stars traveling faster than expected!
⇒ missing mass in galaxies
Two main ideas to explain this
MACHOS- Massive Compact Halo Objects (Astronomical solution)
WIMPS- weakly interacting massive particles (particle physics solution)
~4% of universe is "ordinary matter" (i.e. baryonic)
~25% is dark matter
~70% is dark energy
Dark energy
After big bang gravity acts to slow expansion
Observations of type IA supernova (standard candles) show for a particular red shift (relative speed) have to look at fainter objects than predicted ⇒ the earth is further away than expected⇒ implies universe (at large distances) is expanding faster than predicted
? Big question
Gravity/GUT/TOE/String Theory
Std. model doesn't include gravity
GUT- Grand Unified Theory
=Strong +Electro-weak
TOE- Theory of Everything
=GUT+Gravity
String Theory
4. Expt Methods
Accelerators
Either/or

Fixed Target	Colliding beams
→ |	→←

Either linear or cyclic (circular)
Fixed target vs collider
Recall W=invariant mass^2=amount of energy available to prroduce new particle
(15) → $W \sim \sqrt{E_{bean}}$ fixed target
$W \sim E_{beam}$ collider
So collider preferred
Linear cf Cathode Ray Tube
Cyclic
a) Cyclotron (Lawerence 1929)
Charged particle move in a circle in a mag fld $\underline{B}$ and are accelerated once/twice per orbit
⇒ for a fixed $\underline{B}$ they''ll spiral outwards
Since particle is moving in a spiral ⇒ accelerating⇒ emits radiation (photons)⇒ looses energy
Turns out energy loss
$$\sim \frac{1}{m^4}$$
So energy loss for e^- huge cf P

b) Synchrotron (Oliphant)

$\underline{v}$&$\underline{B}$ increase synchronously to maintain const radius

LEP and LHC are synchrotrons