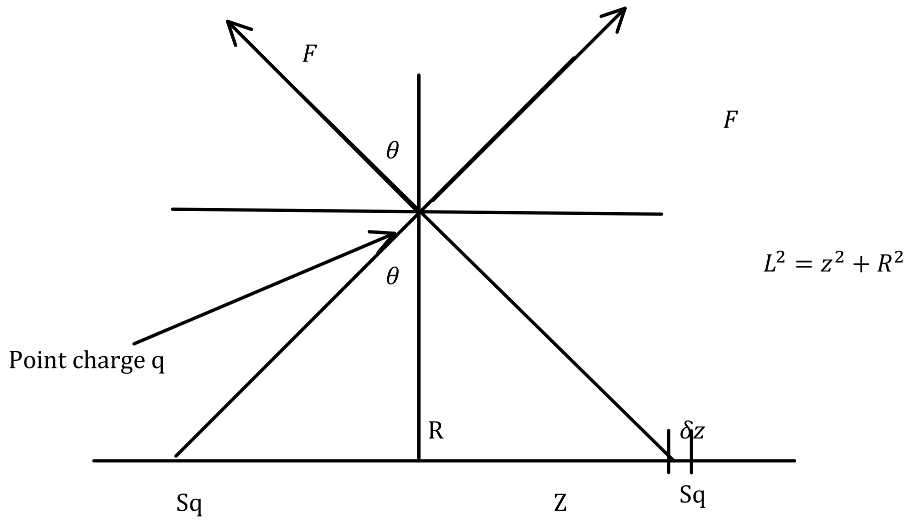
Coulomb's law intro

11 February 2011
15:09

$F = kq_1 q_2 \frac{1}{L^2}$
q=coulomb
$k = \frac{Nm^2}{(coulomb)^2}$
$k = 9 \times 10^9\ Nm^2 C^{-2}$

Superposition Principle
$\int \frac{dz}{(z^2+R^2)^{\frac{3}{2}}} = \frac{z}{R^2\sqrt{Z^2+R^2}}$
$\lambda = linear\ density\ of\ charge$
$\cos\theta = \frac{R}{L}$



$F = \left|\frac{2k\delta q q_{probe}}{L^2}\frac{R}{L}\right|\hat{R} = \left|2kq_{probe}\delta q\frac{R}{L^3}\right|\hat{R}$

$\left.\begin{matrix}\delta q = \lambda dz \\ L = \sqrt{R^2+Z^2}\end{matrix}\right\} F = \left|2kq_{probe}\lambda\frac{Rdz}{(R^2+Z^2)^{\frac{3}{2}}}\right|\hat{R}$

$F = \left|2kq_{probe}\lambda R\int_0^\infty \frac{dz}{(R^2+Z^2)^{\frac{3}{2}}}\right|\hat{R}$

$F = 2kq_{probe}\lambda\left|\frac{Z}{\sqrt{Z^2+R^2}}\right|_0^\infty = \frac{2kq_{probe}\lambda}{R}\hat{R}$

$\boxed{E = \frac{2k\lambda}{R}\hat{R}}$

$E = -\frac{dV}{dR}\hat{R}$

$\boxed{V = -2k\lambda\log R}$

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$F = \left(\frac{kq\delta q}{R^2}\right)\hat{R}$

$U = \frac{kq\delta q}{R}$

Such that

$-\frac{dv}{dR} = F$

Electric field

$\bar{E} = \frac{kq}{R^2}\hat{R}$

$V = \frac{kq}{R}$

Example

Infinitely long charge distribution

$\int \delta E = \int \left(\frac{2k\delta q}{L^2}\cos\theta\right)\hat{R} = \int_0^\infty 2\frac{k\lambda\eta}{(R^2+L^2)^{\frac{3}{2}}}dz \Rightarrow \boxed{E = \frac{2k\lambda}{R}\hat{R}}$

$-\frac{dV}{dR} = E$

Point charge q

$E = \frac{kq}{R^2}\hat{R}$

$V = \frac{kq}{R}$

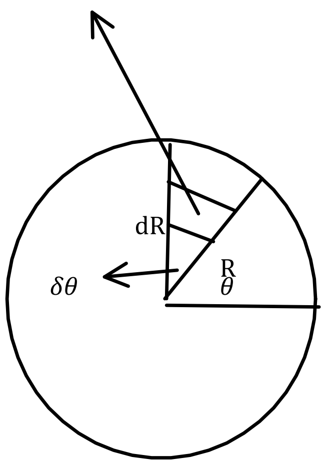
Infinite cable

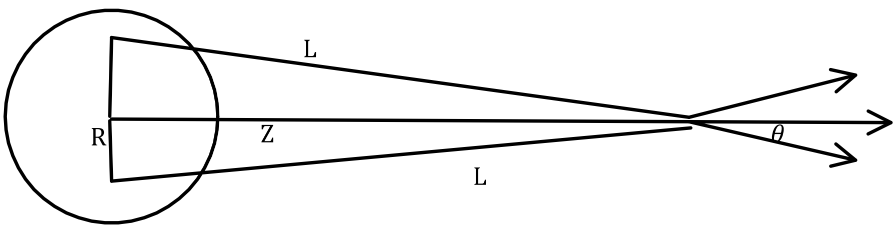
$E = \frac{2k\lambda}{R}\hat{R}$

$V = -2k\lambda \log R + c$

$surface = Rd\theta\delta R$

Area is so small it can be approximated as rectangle





σ (density of charge)

$\delta\bar{E} = \left(\frac{2k\sigma R dR d\phi\cos\theta}{L^2}\right)\hat{K} = \left(2k\sigma\frac{Rz}{(R^2+z^2)^{\frac{3}{2}}}d\phi dR\right)\hat{k}$

$E = 2k\sigma\int_0^\pi\int_0^r \frac{Rz}{(R^2+z^2)^{\frac{3}{2}}}dR\,d\theta = 2k\sigma\pi\int_0^r\frac{Rz}{(R^2+z^2)^{\frac{3}{2}}}dR$

$E = 2k\sigma\pi z.\left|-\frac{1}{\sqrt{R^2+z^2}}\right|_0^R = 2k\sigma\pi z\left|\frac{1}{\sqrt{R^2+z^2}}+\frac{1}{z}\right| = 2k\sigma\pi\left|\frac{z}{\sqrt{R^2+z^2}}+1\right|$

$\bar{E} = 2k\sigma\pi\left(1-\frac{z}{\sqrt{R^2+z^2}}\right)\hat{k}$

What happens if $R \to \infty$?

$E = 2k\pi\sigma\,\hat{k}$

Infinite plane of carge

What happens if $R \to 0$

$\bar{E} = \frac{\#}{Z^2} \to take\ a\ look\ in\ notes$

Infinite plane

$\bar{E} = 2\pi\sigma k\ \hat{k}$

$V = -2\pi k\sigma z + \hat{C}$

Point charge

Point charge	$E \sim \frac{1}{R^2}$	$V \sim \frac{1}{R}$
Infinite line	$E \sim \frac{1}{R}$	$V \sim \log R$
Plane	$E \sim 1$	$V \sim R$
Volume	$E \sim R$	$V \sim R^2$

$E \sim \frac{1}{R^{D-1}}$

D=number of dimensions

4 dimensions

Point charge

$$E \sim \frac{1}{R^3}$$

Lisa Randall

FIRST MIDTERM FEB 25

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10:04

Flux of $\bar{v}$ on surface S

$$\int_{surface} \bar{v}.d\bar{s}$$

$$\int \bar{v}.d\bar{s} = \int |v|.|ds|(\bar{R}.\bar{R}) = |\bar{v}|\int ds = |v|.4\pi R^2$$

$\bar{v} = |v|.\hat{R}$
Velocity is in radial direction
$d\bar{s} = ds_1\hat{R} + ds_2\hat{K} - ds_3\hat{K}$
K is along axes

$$\int \bar{v}.d\bar{s} = (v.\hat{R})\left(ds_1\hat{R} + \boxed{\begin{matrix} ds_2\bar{K} - ds_3\bar{K} \\ cancels \end{matrix}}\right)$$

Gauss' law

$E = \frac{Kq}{R^2} \Rightarrow E4\pi R^2 = Kq4\pi$

$4\pi R^2 \Rightarrow surface\ area\ of\ s^2$

$\int_{s^2} \bar{E}.d\bar{s} = kq4\pi$

$\bar{E}2\pi Rz = 2k\lambda z2\pi$

$\int_{cylinder} \bar{E}.d\bar{s} = 2\pi kq$

$\pi R^2 E = 2.2k\sigma\pi R^2 = 4\pi kQ$

$\int_{closed\ surface} \bar{E}.d\bar{s} = 2\pi kq_{enclosed\ by\ the\ surface}$

$\rightarrow gauss'law$

$$\oint_{\Sigma} \bar{E}.d\bar{s} = 2\pi k$$

Review

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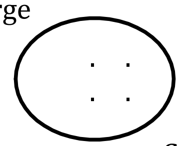
13:05

Most important result of previous lectures-> Gauss' theorem
Distribution of electric charge



Closed surface Σ

$$\oint_{\Sigma} \bar{E}.d\bar{s} = Q_{enclosed} 4\pi\kappa = \frac{Q_{enclosed}}{\epsilon_0}$$

$d\bar{s}$ is a normal vector from each point on the surface

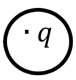
$$\kappa = \frac{1}{4\pi\epsilon_0}$$
$$\epsilon_0 = electric\ permativity\ of\ vacuum$$

Observations:

1. If there are no charges inside
   $$\oint_{\mathrm{E}} \bar{E}.d\bar{s} = 0$$

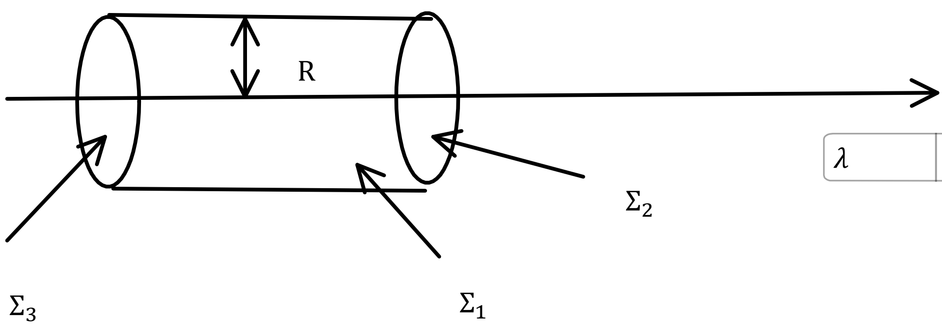
2. $$\oint_{\mathrm{E}} \bar{B}.d\bar{s} \equiv 0$$
   There are no isolated magnetic charges- if a magnet is cut in two, you get two magnets; poles cannot be separated

$$\bar{E} = \frac{\kappa q}{R^2}\hat{R}$$



Point charge q

$$\bar{E} = |E| \cdot \hat{R}$$
$$d\bar{s} = |ds|\hat{R}$$
$$\bar{E}d\bar{s} = |E||ds|\hat{R}\hat{R}$$
$$\hat{R}\hat{R} = 1$$
$$\oint_{\mathrm{E}} \bar{E}d\bar{s} = \int_{\mathrm{E}} |E||ds| = |E| \int_{\mathrm{E}} |ds| = |E|.surface\ of\ sphere$$
$$E.(4\pi R^2) = q \rightarrow \bar{E} = \frac{q}{4\pi\epsilon_0 R^2}\hat{R}$$



λ	z

$\bar{E} = |\bar{E}|\hat{R}$

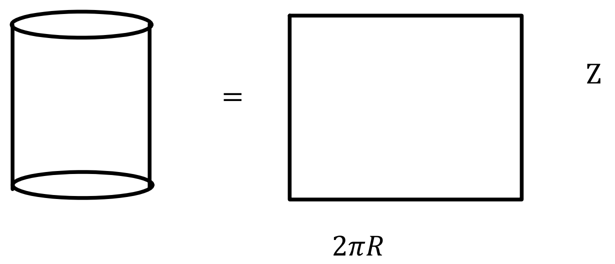
$d\bar{s} = |d\bar{s}_I|\hat{R} + |d\bar{s}_{II}|\hat{R} + |d\bar{s}_{III}|(-\hat{R})$

$\bar{E}(d\bar{s}_I + d\bar{s}_{ii} + d\bar{s}_{iii})$

$|E||ds_i|\hat{R}.\hat{R} + |E||ds_{ii}|\hat{R}.\hat{R} + |E||ds_{iii}|(-\hat{R}.\hat{R}) = |E||ds_i|\hat{R}.\hat{R}$

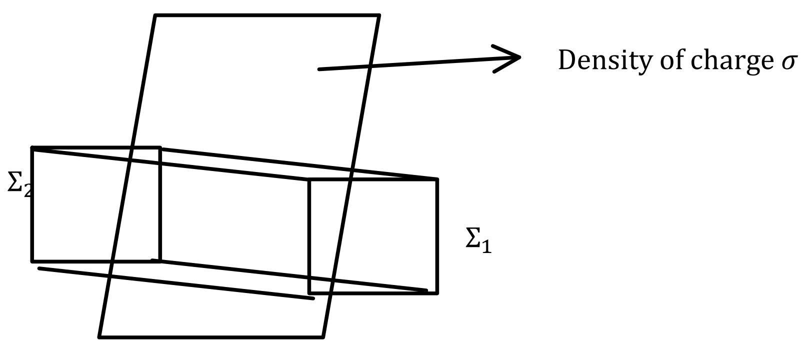
$\bar{E}d\bar{s} = |E||ds_i|$

$\int_{\Sigma} \bar{E}d\bar{s} = \int_{\Sigma_1} \bar{E}d\bar{s}_1 = |E| \int_{\Sigma_1} ds_1$



$$= E.2\pi R.Z = \lambda.\frac{Z}{\epsilon_0}$$

$$\bar{E} = \left(\frac{\lambda}{2\pi E_0}\frac{1}{R}\right)\hat{R}$$



$$\oint_{\Sigma} \bar{E}d\bar{s} = \int_{\Sigma_1} \bar{E}d\bar{s} + \int_{\Sigma_2} \bar{E}d\bar{s}$$

Other surfaces parallel to electric field, therefore 0

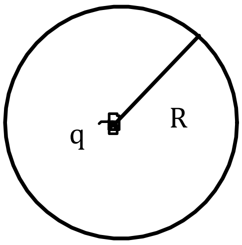
$$\int |E||ds|\,\hat{R}\hat{R} + |(-\hat{R})$$

$$|E| \int_{\Sigma_1} |ds_1| + |E| \int_{\Sigma_2} |ds_2| = |E|x.y + |E|x.y = 2|E|x.y = \frac{\sigma_0 xy}{\epsilon_0}$$

For a closed surface enclosing charges Q

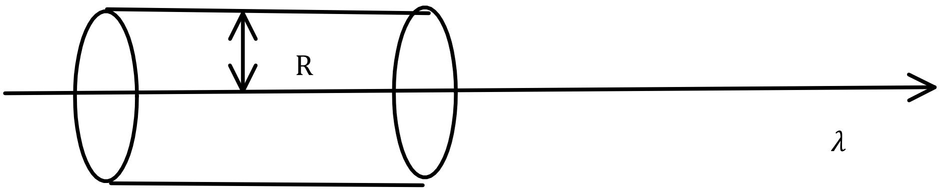
$$\oint_{\Sigma} \bar{E} d\bar{s} = \frac{Q}{\epsilon_0}$$

$$Surface = 4\pi R^2$$

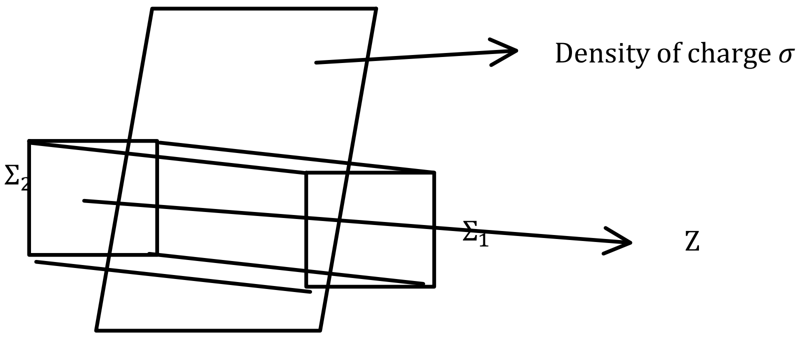


$\hat{R}$- normal vector

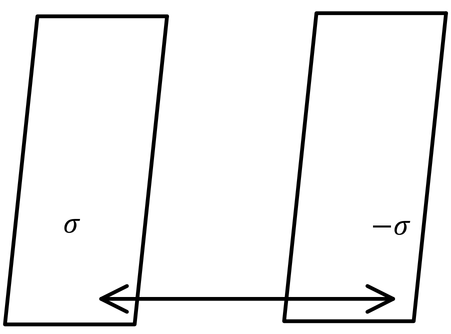
$$\bar{E} = \frac{q}{4\pi\epsilon R^2}\hat{R}$$

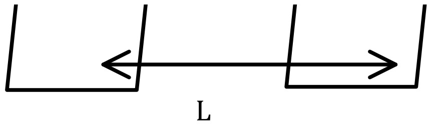


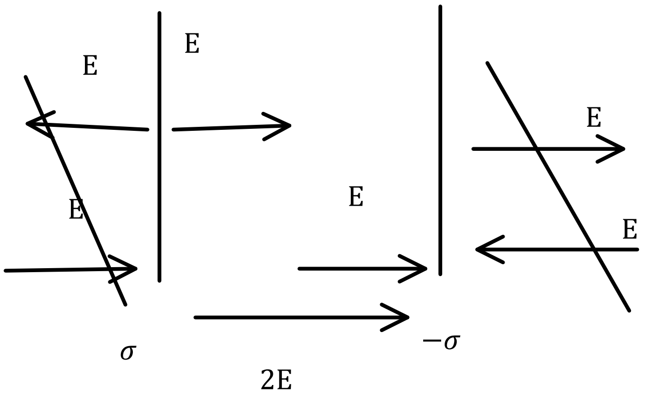
$$\bar{E} = \frac{\lambda}{2\pi\epsilon_0 R}\hat{R}$$



$$\bar{E} = \begin{cases} \frac{\sigma}{2\epsilon_0}\hat{R}\ z > 0 \\ \frac{\sigma}{2\epsilon_0}\ \left(-\hat{R}\right) z < 0 \end{cases}$$

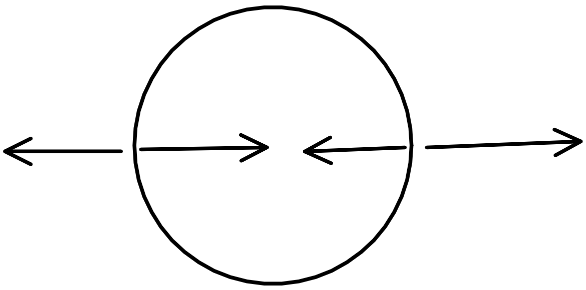






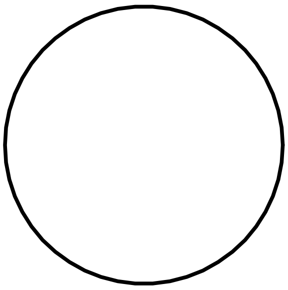
$$E = \frac{2\sigma}{2\epsilon_0}\hat{R} = \frac{\sigma}{\epsilon_0}\hat{R}$$

If you charge a conductor, all the charge flows to the edges



Conducting sphere with charge q

$$\bar{E} = \begin{cases} \frac{q}{4\pi\epsilon R^2}\hat{R} \quad \hat{R} & z > 0 \\ 0 \ \left(-\hat{R}\right) & z < 0 \end{cases}$$



Insulating sphere

Electric fiedl outside

$\bar{E} = \frac{q}{4\pi\epsilon R^2}\hat{R}$

Total charge

$$Q = \frac{4\pi}{3} r R^3 \rho$$

$$\bar{E} = \begin{cases} \bar{E} = \frac{\rho}{3\epsilon_0} R \quad \hat{R} \ \ inside\ sphere \\ \\ \bar{E} = \frac{Q}{4\pi\epsilon_0}\frac{1}{R^2}\ \hat{R}\ \ outside\ sphere \end{cases}$$

$$\int \bar{E} d\bar{s} = \int |E||ds|\hat{R}\hat{R} = |E| 4\pi R^2 = \frac{1}{\epsilon_0}\rho\frac{4\pi}{3}\frac{R^3}{R^2}$$

$$-\frac{d}{dr}V = E$$

$$\bar{\nabla} f = \frac{\delta f}{\delta x}\hat{\imath} + \frac{\delta f}{\delta y}\hat{\jmath} + \frac{\delta f}{\delta z}\hat{k}$$

$$\bar{F} = -\frac{dU}{dz}\hat{R} = -\bar{\nabla} V$$

Electric potential=the work done by the electrical field on a charge that we move to the point "R"

Magnetism->historical development->we will not follow

Lorentz->force on a charged particle

$$\boxed{\bar{f} = q\bar{E} + q\bar{v} \times \bar{B}}$$

$$\bar{f} = q v_0 \hat{\imath} \times B_0 \hat{k}$$

$$\bar{f} = q\begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ v_0 & 0 & 0 \\ 0 & 0 & B_0 \end{vmatrix} = -q v_0 B_0 \hat{\jmath}$$

What is the work (change of kinetic energy) done by the magnetic field

$$\delta w = \bar{f}.d\bar{R}$$

$$\delta w = q(\bar{v} \times \bar{B}).d\bar{R} = q\left(\frac{d\bar{R}}{dt} \times \bar{B}\right).d\bar{R}$$

$$\bar{f} = mAe.vB = \bar{I} \times \bar{B} \equiv I.\bar{l} \times \bar{B} \rightarrow$$

$$q\begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ v_x & v_y & v_z \\ 0 & 0 & B_0 \end{vmatrix} = \hat{\imath} q(v_y B_0) - \hat{\jmath} q(v_x B_0)$$

$$\bar{f} =, \bar{a} = m\frac{d\bar{v}}{dt} = m\left(\frac{d\bar{v}_x}{dt}\hat{\imath} + \frac{d\bar{v}_y}{dt}\hat{\jmath} + \frac{d\bar{v}_z}{dt}\hat{k}\right)$$

$$m\frac{dv_x}{dt} = qB_0 v_y$$

$$m\frac{dv_y}{dt} = -qB_0 v_x$$

$$m\frac{dv_z}{dt} = 0$$

$$\frac{dv_x}{dt} = \left(\frac{qB}{m}\right)\bar{v}_y \qquad (1)$$

$$\frac{dv_y}{dt} = \left(\frac{qB}{m}\right)\bar{v}_x \quad (2)$$

Take equation one and derivate it

$$\frac{d^2v_x}{dt^2} = \frac{qB}{m}\frac{d}{dt}v_y = \frac{d^2v_x}{dt^2} = \left(\frac{qB}{m}\right)\left(-\frac{qB}{m}\right)v_x$$

$$\frac{d^2v_x}{dt^2} = -\left(\frac{qB}{m}\right)^2 v_x$$

Doing viconverce

$$\frac{d^2v_y}{dt} = -\left(\frac{qB}{m}\right)v_y$$

$$v'' = -\omega^2 v$$

$$v_x = A_x \cos(\omega t + \rho_x)$$

$$v_y = A_y \cos(\omega t + \rho_y)$$

Qt t=0

$$\bar{v} = v_0\hat{\imath}$$

$$x_x(t=0) = A_x \cos\phi_x = v_0$$

$$v_y(t=0) = A_y \cos\phi_y = 0$$

Choose

$$\phi_x = 0 \quad A_x = v_0$$

$$\phi_y = \frac{\pi}{2}$$

$$v_z = v_0 constant$$

$$v_x = v_0 \cos\omega t$$

$$v_y = A\cos\left(\omega t + \frac{\pi}{2}\right)$$

$$z = z_0 + v_{0z}t$$

$$x = x_0$$

We started with magnetic fields that we "defined" via the force on a charged moving particle

$$\bar{f} = q\bar{E} + q\bar{v} \times \bar{B}$$

We offered this for a large set of particles that are moving in side a cable/wire

$$\bar{f} = (mAev)d\bar{l} \times \bar{B}$$

$$mAev = electric\ current$$

$$\Rightarrow \left[\bar{f} = Id\bar{l} \times \bar{B}\right]$$

We showed that the magnetic force does not change the kinetic energy of the particle [does not work]

$$f = qvB$$

$$Newton = Kg\frac{m}{s^2} = coulomb.\frac{m}{s}[B] \rightarrow [B] = \frac{kg}{coulomb.\sec} = \text{Tesla}$$

$$f_{magnetic} = qv_0B = m\frac{v_0^2}{R}$$

$$R = \frac{mv}{qB}$$

$$\bar{g} = -\nabla(gz) = -g\hat{k}$$

$$\bar{E} = -\left(\frac{d}{dx}V\hat{\imath} + \frac{d}{dy}V\hat{\jmath} + \frac{d}{dz}V\hat{k}\right) = -\bar{\nabla}V$$

$$\int_b \bar{f}d\bar{R} \Rightarrow q\int_b \bar{E}d\bar{R}\,;\ \oint_b \bar{E}d\bar{R} = 0$$

If

$$\bar{E} = -\frac{d}{dx}V$$

$$\oint_b \bar{B} d\bar{l} = \mu_0 I$$

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We saw

Lorentz force

$\bar{f} = q\bar{E} + q\bar{v} \times \bar{B}$

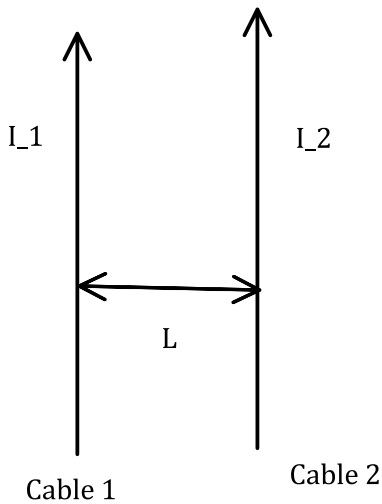
$\bar{f}' = q\bar{E} + q\bar{l} \times \bar{B}$

And we saw

$$\oint_b \bar{B}.d\bar{l} = \mu_0 I$$

$$\delta\bar{B} = \frac{I.d\bar{l} \times \bar{R}}{|\bar{R}|^3}$$

$$\bar{B} = \frac{\mu_0 I}{2\pi R}\hat{\phi}$$



Compute the force of cable 1 on cable 2

$\bar{f} = Id\bar{l} \times \bar{B}$

1) Compute the magentic field that the cable 1 produces in the portion of the cable 2 $\bar{B}_{1\to 2}$
2) Compute the force $\bar{f} = I_2 d\bar{l}_2 \times \bar{B}_{1\to 2}$

1) $\bar{B}_{1\to 2} = \frac{\mu_0 I_1}{2\pi L}\hat{\phi}$
2) $\bar{f} = I_2 d\bar{l}_2 \hat{k} \times \bar{B}_{12}$

   $\bar{f} = I_2 dl_2 \hat{k} \times \frac{\mu_0 I_1}{2\pi L}\hat{\phi}$

   $\bar{f} = \frac{\mu_0 I_1}{2\pi L} dl_2 (\hat{k} \times \hat{\phi})$

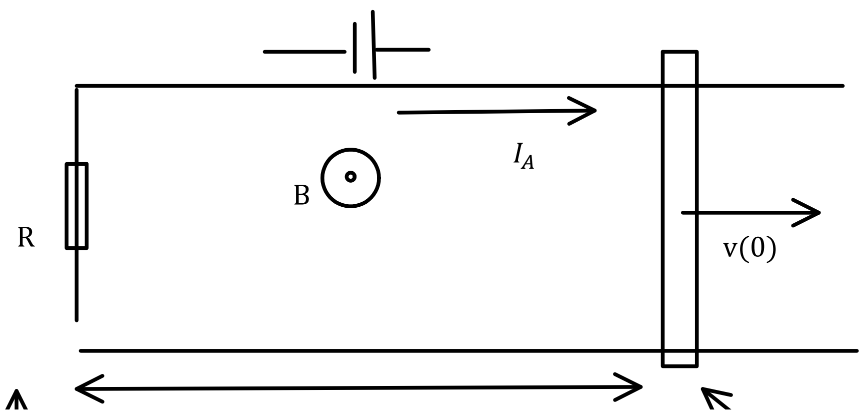
Faraday's law

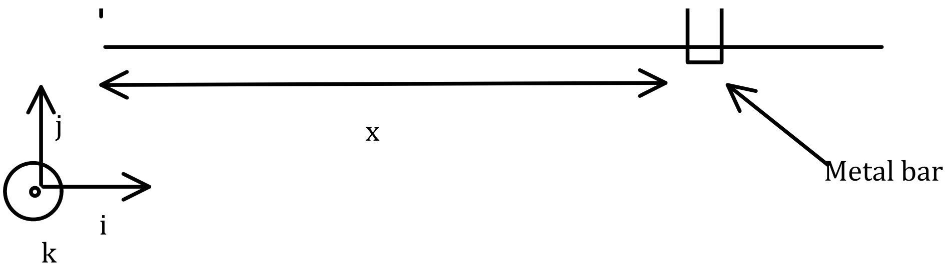
$$\frac{d}{dt}\int_{\bar{z}} \bar{B} d\bar{s} = -\oint_b \bar{E} d\bar{l}$$

$$\Rightarrow \frac{d}{dt}\Phi_B = \epsilon$$

$$\int_{\Sigma} \bar{B} d\bar{s} = \Phi_B$$

$= flux\ of\ the\ magnetic\ field\ on\ surface\ \Sigma$





Flux of the magnetic field

$\bar{B} = B_0\hat{k}$

$d\bar{s} = ds\hat{k}$

$\bar{B}.d\bar{s} = B_0 ds$

$\int_\Sigma \bar{B}d\bar{s} = \int_\Sigma B_0 ds = B_0 \int_\Sigma ds = B_0\Sigma = B_0 L.x = \Phi_B$

Σ =total surface

$\Phi = B_0 L\, x(t)$

$\boxed{\frac{d\Phi}{dt} = B_0 L\dot{x}(t)}$ $\boxed{\epsilon = -\frac{d}{dt}\Phi = -B_0 L\dot{x}(t)}$

- Electric field opposing current

$\epsilon = V = B_0 L\dot{x}$

$\epsilon V = RI$

$I = \frac{V}{R} = \frac{B_0 L\dot{x}}{R}$

We have a current (charged particles moving) in a magnetic field

$\bar{f} = I\bar{L} \times \bar{B} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 0 & -Il & 0 \\ 0 & 0 & B_0 \end{vmatrix} = -ILB_0(\hat{\imath})$

$\bar{B} = B_0\hat{k}, \qquad I\bar{L} = IL(-\hat{\jmath})$

$I = \frac{B_0 L}{R}\dot{x}$

$\boxed{\bar{f} = -\frac{L^2 B_0^2}{R}\hat{\imath}}$

$\bar{f} = m\bar{a}$

$f = m\ddot{x}$

$\boxed{-\frac{L^2 B^2}{R}\dot{x} = m\ddot{x}}$

Type equation here.

Summary of things for exam

31 March 2011
13:12

Gauss' law

$$\oint_\Sigma \bar{E}.d\bar{s} = \frac{\sigma}{\epsilon_0}$$

$\sigma = total\ electric\ charge\ articulated\ by\ \Sigma$
$\epsilon_0 = electric\ permitivity\ of\ vacuum$

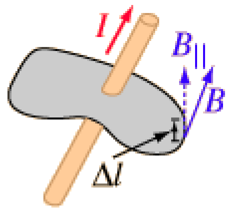
$$\oint_\Sigma \bar{B}d\bar{s} = 0$$

Eg impossible to have monopole magnet

Faraday law

$$-\frac{d}{dt}\int_{\Sigma_1} \bar{B}d\bar{s} = \epsilon = \oint_b \bar{E}d\bar{l}$$

ϵ =induced voltage->induced emf
Ampere's law
Open surface, around a current



$$\sum B_{||}\Delta l = \mu_0 I$$

$$\oint_b \bar{B}d\bar{l} = \mu_0 I$$

μ_0 = magnetic permittivity of vacuum
I=total current passing through the surface Σ_1

Maxwell realized (~1850) that Ampere's law was incomplete. He called the term
$\frac{d}{dt}\int_{\Sigma_0} \bar{E}d\bar{s}$ →"displacement current

$$\oint_b \bar{B}d\bar{l} = \mu_0 I - \mu_0\epsilon_0\frac{d}{dt}\int_{\Sigma_0} \bar{E}d\bar{s}$$

If he added this term->many interesting things werehappening with these eqs
What things do happen?

You can derive a "wave equation" for electric and magnetic

$$\frac{d^2}{dx^2}\bar{E} = \frac{1}{c^2}\frac{d^2\bar{E}}{dt^2}$$

Or same B; interestingly, $\boxed{c^2 = \mu_0\epsilon_0}$

$$\frac{d^2}{dx^2}\Psi = \frac{1}{c^2}\frac{d^2}{dt^2}\Psi$$

Electric charge was conserved

Maxwell's Equations

$$\oint_\Sigma \bar{B}d\bar{s} = 0$$

$$\oint_{\Sigma} \bar{E} d\bar{s} = \frac{\sigma}{\epsilon_0}$$
$$-\frac{d}{dt}\int_{\Sigma_1} \bar{B} d\bar{s} = \oint_b \bar{E} d\bar{l}$$
$$\oint_b \bar{B} d\bar{l} = \mu_0 I + \mu_0 \epsilon_0 \frac{d}{dt}\int_{\Sigma_1} \bar{E} d\bar{s}$$
$\Sigma_1 =$ open surface with boundary b

Exercise: 1. take the Maxwell's equations and put all their charges and currents=0

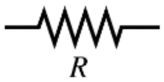
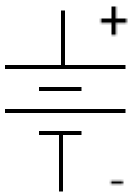
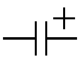
$Q = I = 0$

2. Next, what happens if you exchange $\bar{E} \to \bar{B}, \bar{B} \to -\bar{E}$

$$\oint_{\Sigma} \bar{E} d\bar{s} = 0$$
$$\oint_b \bar{B} d\bar{l} = \mu_0 \epsilon_0 \frac{d}{dt}\int_{\Sigma_1} \bar{E} d\bar{s}$$
$$\oint_{\Sigma} \bar{E} d\bar{s} = 0$$
$$\frac{d}{dt}\int \bar{E} d\bar{s} = \oint \bar{B} d\bar{l}$$

Electric circuits

05 May 2011
13:07

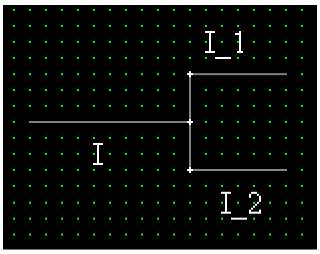
	Units	Difference of portential
 Resistors	Ohm Ω	$V_{ab} = RI$
 Battery	Volts	$V_{ab} = -V$
 capacitor	Faraday $\mu f, pf$	$V = \frac{Q}{c}$
Currents	Ampere	
 Inductor		$V = L\frac{dI}{dt}$

Ohm's Law

$$\boxed{V = RI}$$

Difficult to get- hard to measure

Kirchhoff's law



$I = I_1 + I_2$

Power dissipated

$$P_{dissipated} = RI^2$$

$$I = \frac{V}{R} = \frac{V}{\left(\left(\frac{1}{R_2} + \frac{1}{R_2} + \frac{1}{R_3}\right)^{-1} + R_1\right)}$$

$$Power\ dissapated = RI^2 = pu\left(\left(\frac{1}{R_2}+\frac{1}{R_2}+\frac{1}{R_3}\right)^{-1}+R_1\right)\left(\frac{V}{\left(\left(\frac{1}{R_2}+\frac{1}{R_2}+\frac{1}{R_3}\right)^{-1}+R_1\right)}\right)^2$$

$$= \frac{V^2}{\left(\left(\frac{1}{R_2}+\frac{1}{R_2}+\frac{1}{R_3}\right)^{-1}+R_1\right)}$$

$R_k = k^2$
$k \in \mathbb{Z}$
$$\sum_{k=0}^{\infty}\frac{1}{R_k} = finite\ number$$

Capacitor connected to resistor

$$\left[\frac{Q}{c}+RI=0 \rightarrow \frac{dQ}{dt}+\frac{Q}{Rc}=0\right]$$

$$\left[\frac{dQ(t)}{dy}+\frac{Q(t)}{Rc}=0\right]$$

2 ways to solve

1st way

$$\frac{dQ}{dt} = -\frac{Q}{Rc} \rightarrow \frac{dQ}{Q} = -\frac{dt}{Rc} \rightarrow integrate$$
$$\int\frac{dQ(t)}{Q(t)} = -\int\frac{dt}{RC} = -\frac{1}{RC}\int dt = -\frac{1}{RC}(t-t_0)$$
$$\left[\log\left(\frac{Q(t)}{c}\right) = -\frac{1}{RC}(t-t_0)\right] \rightarrow Q(t) = ce^{-\frac{t-t_0}{RC}}$$

Solution

$Q(t) = Q_0e^{-\frac{t}{RC}}$

2nd way

Propose

$Q = Ae^{-\lambda t}$

$$\frac{dQ}{dt} = A\lambda e^{\lambda t}$$

$$A\lambda e^{\lambda t}+\frac{Ae^{\lambda t}}{RC}=0$$
$$Ae^{\lambda t}\left\{\lambda+\frac{1}{RC}\right\}=0$$

$Ae^{\lambda t}$ is non-trivial solution ∴ not zero

$$\lambda = -\frac{1}{RC}$$

Come back

$Q(t) = Ae^{-\left(\frac{t}{RC}\right)}$
$A = Q_0$

So

$$I(t) = -\frac{Q_0}{RC}e^{-\frac{t}{RC}}$$

$Power\ dissapated = RI^2$

$$P = R\left(-\frac{Q_0}{RC}e^{-\frac{t}{RC}}\right)^2 = R\left(\frac{Q_0^2}{R^2C^2}e^{-\frac{2t}{RC}}\right) = \frac{Q_0^2}{RC^2}e^{-\frac{2t}{RC}}$$

Total power =

$$\int_0^\infty Pdt = \int_0^\infty \frac{Q_0^2}{RC^2}e^{-\frac{2t}{RC}}dt = \frac{Q_0^2}{RC^2}\int_0^\infty e^{-\frac{2t}{RC}}dt = -\frac{2t}{RC}\frac{Q_0^2}{RC^2}e^{-\frac{2t}{RC}} = \left[-\frac{2tQ_0^2}{R^2C^3}e^{-\frac{2t}{RC}}\right]_0^\infty$$

$$\Rightarrow 0$$

Add inductor to circuit

$$\frac{dQ(t)}{dt} + \frac{Q(t)}{RC} = 0$$

$$L\frac{dI(t)}{dt} + RI(t) + \frac{Q(t)}{c} = 0 \rightarrow \boxed{\frac{Ld^2Q}{dt^2} + \frac{R}{L}\frac{dQ}{dt} + \frac{Q}{LC} = 0}$$

$$I = \frac{dQ}{dt}$$

i) Suppose that we consider R=0

What will happen to the equation?

$$\frac{Ld^2Q}{dt^2} + \frac{Q}{LC} = 0$$

Undamped oscillator

System is the same if t=-t

$$Q(t) = A\sin\left(\frac{1}{\sqrt{LC}}t + \varphi\right)$$

$$\frac{Ld^2Q}{dt^2} + \frac{R}{L}\frac{dQ}{dt} + \frac{Q}{LC} = 0$$

Resistance term=damping

$$\ddot{x} + r\dot{x} + \omega^2 x = 0$$

Damping makes system time dependent $t \neq -t$