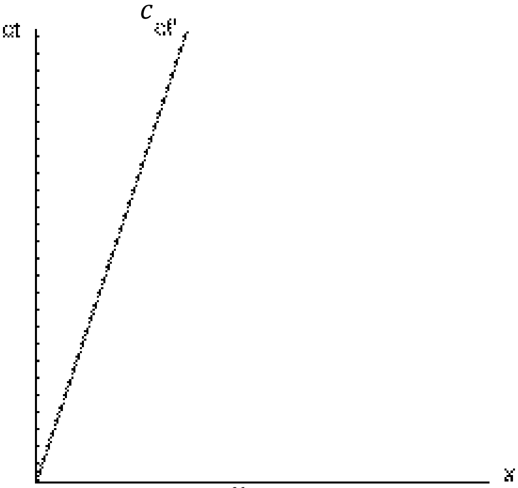
Revision

09 January 2012
11:16

Postulate 1B: Speed of light
1A: Inertial frames equiv

$x' = x - \frac{v}{c}ct$

$ct' = ct - \frac{v}{c}x$



$$\Rightarrow \frac{x'}{t'} = \frac{x - vt}{t - \frac{vx}{c^2}} = \frac{\frac{x}{t} - v}{1 - \frac{v\frac{x}{t}}{c^2}} \Leftrightarrow u' = \frac{u - v}{1 - \frac{vu}{c^2}}$$

u=c

$$u' = \frac{c - v}{1 - \frac{v}{c}} = c$$

$$\begin{cases} x' = \gamma\left(x - \frac{v}{c}ct\right) \\ ct' = \gamma\left(ct - \frac{v}{c}x\right) \end{cases}$$

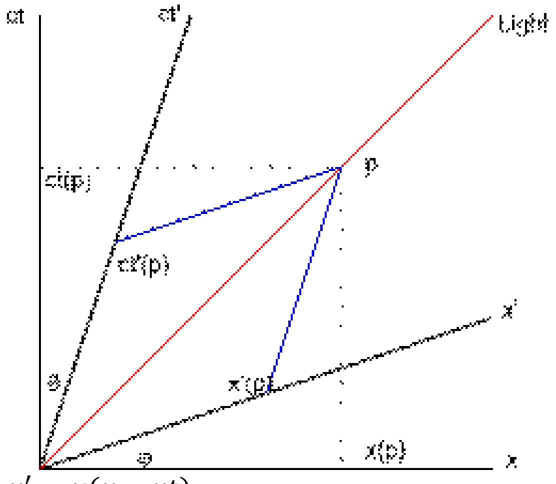
$$\Rightarrow \begin{cases} x = \gamma\left(x' - \frac{v}{c}ct'\right) \\ ct = \gamma\left(ct' - \frac{v}{c}x'\right) \end{cases}$$

$$\Rightarrow x' = \gamma\left(\gamma\left(x' - \frac{v}{c}ct'\right) - \frac{v}{c}\gamma\left(ct' - \frac{v}{c}x'\right)\right) = \gamma^2\left(\left(x' - \frac{v}{c}ct'\right) - \frac{v}{c}\left(ct' - \frac{v}{c}x'\right)\right)$$

$$= \gamma^2\left(1 - \frac{v^2}{c^2}\right)x'$$

$$\gamma^2\left(1 - \frac{v^2}{c^2}\right) = 1$$

$$\Rightarrow \gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$



$x' = \gamma(x - vt)$

$\Rightarrow x' = 0 \Rightarrow x = vt$

$$\tan\theta = \frac{x}{ct} = \frac{v}{c}$$

$$ct' = \gamma\left(ct - \frac{v}{c}x\right)$$

$\Rightarrow t' = 0 \Rightarrow ct = \frac{v}{c}x$

$$\tan\phi = \frac{ct}{x} = \frac{v}{c}$$

$$\theta = \phi$$

A3.

$$ds^2 = -c^2dt^2 + dx^2 + dy^2 + dz^2$$

Invariant

$s^2 = -c^2t^2 + x^2 + y^2 + z^2$

Claim

$$S^2 = -c^2t'^2 + x'^2 + \cdots$$

$$= -\gamma^2\left(ct - \frac{v}{c}x\right)^2 + \gamma^2(x - vt)^2 + y^2 + x^2$$

$$= \gamma^2\left(-c^2t^2 + 2\frac{v}{c}ctx - \frac{v^2}{c^2}x^2 + x^2 - 2vxt + v^2t^2\right) + y^2 + z^2$$

$$= \gamma^2\left(-c^2t^2 - \frac{v^2}{c^2}x^2 + x^2 + v^2t^2\right) + y^2 + z^2$$

A2.

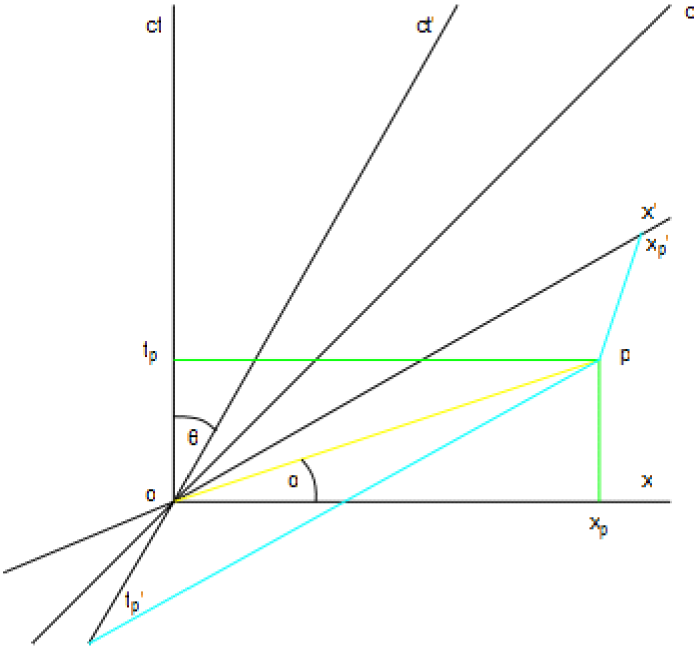
$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} = 7.1$$

t=lifetime in rest frame=26nsec

t'= " earth

$\Leftrightarrow t' = \gamma t = 7.1 \times 26 nsec$

Distance =vt'= $0.99 \times 3 \times 10^8 \times 7.1 \times 26 \times 10^{-9} = 54.8\, m$



0p is a faster than light path

In frame S, op is forwards in time $t_p > t_0$

In frame S', 0P is backwards in time, $t_p' < t_0'$

$\Rightarrow$ If a faster than light signal is possible, Then in some frames it is forwards in time, but in other frames it is backwards in time

$\tan\theta = \frac{v}{c}$

$\tan\alpha = \frac{c}{u}$

Where u is the speed of the tachyon

Backwards in time in S' if the relativity velocity of the frames satisfies

$$\tan\theta > \tan\alpha$$

$$\Rightarrow \frac{v}{c} > \frac{c}{u} \Rightarrow v > \frac{c^2}{u}$$

Directly from Lorentz transformation

For 0p,

$$ct' = \gamma\left(ct - \frac{v}{c}x\right)$$

$$x' = \gamma\left(x - \frac{v}{c}ct\right)$$

Speed of tachyon $u = \frac{x}{t}$

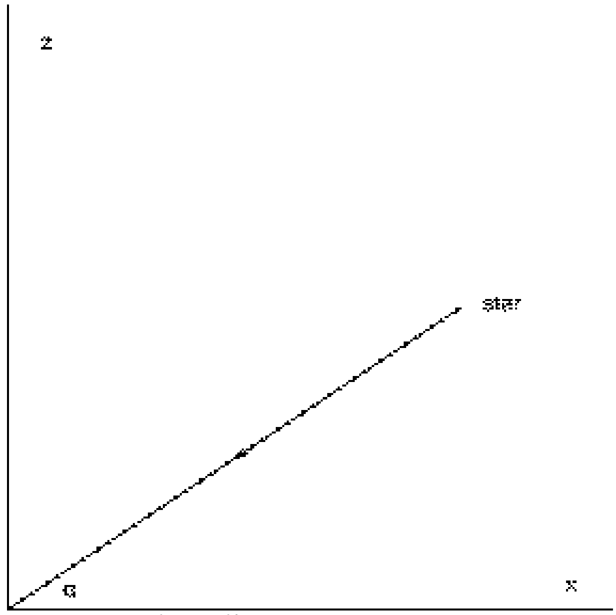
$t' < 0\ if\ ct - \frac{v}{c}x < 0$

$$\Rightarrow ct\left(1 - \frac{v}{c^2}\frac{x}{t}\right) < 0$$

$$\left(1 - \frac{v}{c^2}\frac{x}{t}\right) \Rightarrow 1 - \frac{v}{c^2}u$$

$$\Rightarrow v > \frac{c^2}{u}$$

B1: Stellar aberration



$$u'_x = \frac{u_x - v}{1 - \frac{u_x v}{c^2}}$$
$$u'_y = \frac{u_y}{\gamma(u)\left(1 - \frac{u_x v}{c^2}\right)}$$
$$u_x = -c\cos\alpha \Rightarrow -c\cos\alpha' = u'_x = \frac{u_x - v}{1 - \frac{u_x v}{c^2}}$$
$$u_z = -c\sin\alpha \Rightarrow -c\sin\alpha' = u'_z = \frac{u_z}{\gamma(v)\left(1 - \frac{u_x v}{c^2}\right)}$$
$$\tan\frac{\alpha'}{2} = \frac{\sin\alpha'}{1+\cos\alpha'}$$
$$= \frac{\sin\alpha}{\gamma\left(1+\frac{v}{c}\cos\alpha\right)} \frac{1+\frac{v}{c}\cos\alpha}{\left(1+\frac{v}{c}\right)(1+\cos\alpha)}$$
$$= \frac{1}{\gamma\left(1+\frac{v}{c}\right)}\frac{\sin\alpha}{1+\cos\alpha} = \tan\frac{\alpha}{2}\sqrt{\frac{1-\frac{v}{c}}{1+\frac{v}{c}}}$$
$$\frac{1}{\gamma\left(1+\frac{v}{c}\right)} = \frac{\sqrt{1-\frac{v^2}{c^2}}}{1+\frac{v}{c}} = \frac{\sqrt{\left(1-\frac{v}{c}\right)\left(1+\frac{v}{c}\right)}}{1+\frac{v}{c}} = \frac{\sqrt{1-\frac{v}{c}}}{\sqrt{1+\frac{v}{c}}}$$

$x^\mu = (ct, \underline{x}) = (ct, x, y, z)$

$P^\mu = \left(\frac{E}{c}, \underline{p}\right) = \left(\frac{E}{c}, p_x, p_y, p_z\right)$

LTs

$ct' = \gamma\left(ct + \frac{vx}{c}\right)$

$x' = \gamma\left(x - \frac{v}{c}tc\right)$

y'=y

z'=z

$\frac{E'}{c} = \gamma\left(\frac{E}{c} - \frac{v}{c}p_x\right) \Leftrightarrow E' = \gamma(E - v\,p_x)$

$P'_x = \gamma\left(p_x - \frac{v}{c}\frac{E}{c}\right) \Leftrightarrow P'_x = \gamma\left(P_x - \frac{vE}{c^2}\right)$

$p'_y = p_y$

$p'_z = p_z$

$P = \gamma(u)mu$

$E = \gamma(u)mc^2$

Particle vel u in x-direction

B2(b)

$$E' = \gamma(v)(\gamma(u)mc^2 - v\gamma(u)mu)$$
$$= \gamma(v)\gamma(u)mc^2\left(1 - \frac{uv}{c^2}\right)$$
$$u' = \frac{u-v}{1-\frac{uv}{c^2}}$$
$$\gamma(u') = \gamma(u)\gamma(v)\left(1 - \frac{uv}{c^2}\right)$$
$$= \gamma(u')mc^2$$
$$P' = \gamma(v)\left(\gamma(u)mu - \frac{v}{c^2}\gamma(u)mc^2\right)$$
$$= \gamma(v)\gamma(u)m\frac{u-v}{1-\frac{uv}{c^2}}\left(1-\frac{uv}{c^2}\right)$$
$$= \gamma(u')mu'$$

$$S^2 = -c^2t^2 + x^2 + y^2 + z^2$$
Invariant
$$-\frac{E^2}{c^2} + P_x' + P_y' + P_z'$$
Invariant
$$\Rightarrow -\frac{E^2}{c^2} + |\underline{P}|^2 = -m^2c^2$$
$$\Rightarrow \boxed{E^2 - c^{2|\underline{P}|^2} = m^2c^4}$$
$$\Leftrightarrow E' = \gamma(v)(E - vp_x)$$
$$p_x' = \gamma(v)\left(p_x - \frac{vE}{c^2}\right)$$
$$E'^2 - c^2p'^2 = \gamma(v)^2(E-vp)^2 - c^2\gamma(v)^2\left(p - \frac{vE}{c^2}\right)^2$$
$$= \gamma(v)^2\left(E^2 - 2Evp + v^2p^2 - c^2p^2 + 2c^2\frac{vpE}{c^2} - \frac{c^2v^2E^2}{c^2}\right)$$
$$= \gamma(v)^2\left(E^2 + v^2p^2 - c^2p^2 - \frac{c^2v^2E^2}{c^2}\right)$$
$$\gamma^2E^2\left(1-\frac{v^2}{c^2}\right) - c^2p^2\left(1-\frac{v^2}{c^2}\right)\gamma^2$$
$$= E^2 - c^2p^2$$

$$E^2 - c^2p^2$$
$$= \gamma^2m^2c^4 - \gamma^2c^2m^2u^2 = m^2c^4\gamma^2\left(1-\frac{u^2}{c^2}\right) = m^2c^4$$

B3

A

LHC

$$E_{beam} = 3.5TeV = 3500GeV$$
$$E_{cm} = 7TeV$$
$$E_{beam} = \gamma(u)m_pc^2$$
$$\Rightarrow \gamma(u) = 3500 = \frac{1}{\sqrt{1-\frac{u^2}{c^2}}} \Rightarrow \frac{u}{c} = 0.99999996$$

3b

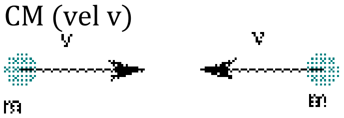
SPS (stationary target)

$$P_1^\mu = \left(\frac{E}{c}, \underline{P}\right)$$
$$p_2' = (mc, \underline{0})$$
$$E_{beam} = 270GeV$$
$$P^\mu_{TOT,LAB} = P_1^\mu + P_2^\mu = \left(\frac{E_{beam} + mc^2}{c}, \underline{p}\right) = -m^2c^2$$

$$P_{tot}^2 = -\frac{(E_{beam} + mc^2)^2}{c^2} + |\underline{P}|^2 = -\frac{E_{Beam}^2}{c^2} + |\underline{P}|^2 - 2E_{beam}m - m^2c^2$$
$$= -2E_{beam}\, m - 2m^2c^2 \approx -2E_{beam}m$$

$$P_{Tot,CM}^{\mu} = \left(\frac{E_{cm}}{c}, \underline{0}\right)$$
$$P_{tot}^2 = \frac{E_{cm}^2}{c^2}$$
KEY step
P_{tot}^2 is invariant
$\Rightarrow -E_{cm}^2 = -2E_{Beam}m$
$E_{cm} = \sqrt{2E_{beam}mc^2}$
$E_{cm} = \sqrt{(2 * 270)} = 23GeV$

2 particle to 2 particle collision
(6.1-compton)
Energy cons
Mom cons
Energy-momentum-mass relation ($E^2 - c^2|\underline{P}|^2 = m^2c^4$)

$$\cos\theta = \frac{E - mc^2}{E + 3mc^2}$$
CM (vel v)


Before

After
Special case of equal angle scattering
$P_x = 0$
$P_y = \gamma(v)mv$
$p_x' = \gamma(v)(p_x + vE)$

Lab	CM
$P_x' =$	$\gamma(v)(P_x + vE)$
$P_y' =$	P_y

$$\Rightarrow \tan\frac{\theta}{2} = \frac{p_y'}{p_x'} = \frac{p_y}{\gamma(v)(P_x + vE)}$$
$$\tan\frac{\theta}{2} = \frac{m}{E} = \frac{1}{\gamma(v)}$$
$$v \to 0, \theta \to \frac{\pi}{2}$$
$$v \to large, \theta \to small$$