PH-203

Assessed Problem Sheet N.1

Date due: Friday November 11, 2011, 10am.

Consider a system of N three-dimensional classical harmonic oscillators. The hamiltonian of the system is $H = \sum_{n=1}^{N} H_n$, with

$$H_n = \frac{1}{2m}\sum_{k=1}^{3} p_{nk}^2 + \frac{\omega^2 m}{2}\sum_{j=1}^{3} q_{nj}^2 ,$$

and the phase-space is given by all possible (q_{nj}, p_{nk}) with $n = 1, \cdots, N$, while $j, k = 1, \cdots, 3$.

a) Using the microcanonical ensemble and the definition

$$\Sigma(E) \equiv \frac{1}{h^{3N}} \int_{H<E} \prod_{ijk} \mathrm{d}q_{ij}\mathrm{d}p_{ik} ,$$

where E is the energy of the system, show that the entropy is

$$S = 3Nk\left(1 + \ln\frac{2\pi E}{3Nh\omega}\right) .$$

[5 Marks]

b) Express the energy of the system in terms of the entropy. [3 Marks]

c) Compute the temperature of the system. [3 Marks]

d) Compute the specific heat C_V. Explain the result in terms of the equipartition theorem. [3 Marks]

e) Express the entropy as a function of the temperature T and number of particles N. [3 Marks]

f) Draw an approximate plot of the entropy as a function of the temperature. What happens when $T \to 0$? Is the third law of thermodynamics reproduced? [3 Marks]

Useful relations

Volume of D-dimensional ball of radius R:

$$V(B^D) = \frac{\pi^{D/2} R^D}{\Gamma_E(1 + D/2)} .$$

Properties of the function Γ_E:

$$\Gamma_E(1+n) = n! \text{ for } n > 0 \text{ integer} .$$

Stirling approximation:

$$\ln N! \simeq N\ln N - N .$$