05 December 2011
18:42

205
Lagrangian → definition
$L = T - V$
where
$T = \frac{m\dot{x}^2}{2}$
V=potential

1. Free Particle → $\boxed{V = 0}$ → $\boxed{L = \frac{m\dot{x}^2}{2}}$
2. Oscillator

$\to V = \frac{Kx^2}{2} \to \boxed{L = \frac{m\dot{x}^2}{2} - \frac{kx^2}{2}}$
F=-kx

Action,
$S = \int_{t_0}^{t} L\, dt$
````

Momentum action principle
impose→ minimize the action $\boxed{\delta S = 0}$
→ find some eqs Euler Lagrange eps

$$\boxed{\frac{d}{dt}\left(\frac{\delta L}{\delta \dot{x}}\right) = \frac{\delta L}{\delta x}}$$

$$\frac{\delta L}{\delta \dot{x}} = \begin{matrix} \to Free & m\dot{x} \\ \to Oscillator & m\dot{x} \end{matrix} \to \frac{d}{dt}(m\dot{x}) = m\ddot{x} + \dot{m}\dot{x}$$

$$\frac{\delta L}{\delta x} = \begin{matrix} \to Free & 0 \\ \to Oscillator & -kx \end{matrix}$$

$Free\ particle \quad m\ddot{x} = 0$
$Oscillator \quad m\ddot{x} = -kx$
Much more complicated if mass ≠ constant

Interference + diffraction
$\psi = \psi_1 + \psi_2$
$P = |\psi_1|^2 + |\psi_2|^2 + 2\psi_1\psi_2$
$\psi_1\psi_2^* + \psi_2\psi_1^*$
$P = P_1 + P_2 + Interference\ term$

211

Energy conservation

$E_1 + mc^2 = E_2 + E_3$
3-momentum conservation
$\underline{P_1} = \underline{P_3} + P_4$
Where $\theta = \theta_3 + \theta_4$
$\Rightarrow |\underline{P_1}|^2 = |\underline{P_3}|^2 + |\underline{P_4}|^2 + 2|\underline{P_3}||\underline{P_4}|\cos\theta$
Consider a special case where the particle separate with equal energies
Assume $E_3 = E_4$
$\Rightarrow \underline{P_3} = \underline{P_4}$
$\Rightarrow \theta_3 = \theta_4$
By conservation of transverse momentum

So the conservation equations simplify
````

$E_1 + mc^2 = 2E_3$ (1)

$E_1^2 - m^2c^4 = 2(E_3^2 - m^2c^4)(1 + cos\,\theta)$ (2)

Using $E_1^2 - c^2|\underline{p}_1|^2 = m^2c^4$ etc

Now solve these to find scattering angle θ as a function of the initial beam energy E_1

Algebra,

$1 \Rightarrow E_1^2 - m^2c^4 = (E_1 + mc^2)(E_1 - mc^2) = 4E_3(E_3 - mc^2)$

Compare 1 and 2

$\Rightarrow 4E_3(E_3 - mc^2) = 2(E_3^2 - m^2c^4)(1 + \cos\theta) = 2(E_3 + mc^2)(E_3 - mc^2)(1 + \cos\theta)$

$\Rightarrow 1 + \cos\theta = \dfrac{4E_3}{2(E_3 + mc^2)}$

$\Rightarrow \cos\theta = \dfrac{E_3 - mc^2}{E_3 + mc^2}$

Finally, substitute for E_3 using $2E_3 = E_1 + mc^2$

$\Rightarrow \boxed{\cos\theta = \dfrac{E_1 - mc^2}{E_1 + 3mc^2}}$

At low energies, $E_1 \approx mc^2$

$\Rightarrow \cos\theta \approx 0 \Leftrightarrow \theta \approx 90°$

So the particles scatter at right angles

this is the well known result in Newtonian dynamics (snooker without spin)

At high energies, $E_1 \gg mc^2$

$\Rightarrow \cos\theta \approx 1 \Leftrightarrow \theta \approx 0$

The higher the energy, the smaller the scattering angle. The particles are scattered into a narrow forward cone. This is a very general result in relativistic scattering.

2+2 scattering in the CM frame

The analysis above was for the lab frame. Now recover the same result in the CM frame

~~

Momenta are equal and opposite in x-direction

Suppose the relative velocity of the CM frame and lab frame is v

$\Rightarrow E_2^{CM} = \gamma(v)m$

Now we showed previously that the total CM energy for fixed-target scattering is

$$E_{CM} = \sqrt{2m(E+m)}$$

$$E_1^{CM} = \frac{1}{2}\sqrt{2m(E+m)}$$

And since $E_1^{CM} = \gamma(v)m$

$\Rightarrow \gamma(v) = \dfrac{1}{2m}\sqrt{2m(E+m)}$

$$\Rightarrow \sqrt{\frac{E+m}{2m}}$$

This determines the velocity v at the CM frame relative to the LAB frame